

Circuit Model Wakefield Computations: Progress Report

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Objectives

- Establish a local capability to compute wakefields
- Improve the efficiency of the computation to allow studies involving an entire linac (10^3) structures.
- Understand the impact of various construction errors (frequency & transverse alignment)
- Do a comparative study of RDDS and MDS

Numerical Computations

Discretization

$$\mathbf{B}(\mathbf{r}) = \mathbf{F}(\mathbf{r}, \alpha_i)$$

$$\mathbf{E}(\mathbf{r}) = \mathbf{G}(\mathbf{r}, \alpha_i)$$

$$\mathbf{B}(\mathbf{r}) = \sum_i \alpha_i \mathbf{F}_i(\mathbf{r})$$

$$\mathbf{H}(\mathbf{r}) = \sum_i \alpha_i \mathbf{G}_i(\mathbf{r})$$

Substitute in differential equation or equivalent weak integral form and solve for α_i .

Circuit Approximation

- The FEM corresponds to a particular choice where F_i and G_i are piecewise continuous interpolation polynomials with localized support. The system of equations is typically very large and very sparse.
- For the Equivalent Circuit Model the approximation basis corresponds to modes of individual cells. Since the transverse beam offset is small, only lowest order modes need be included (TM110, TE111). Instead of dealing with the mode amplitudes directly, the latter are scaled in such a way that the electric and magnetic energy associated with a given mode can be obtained from pseudo voltages and currents.

Coupling

Determination of the Circuit Parameters

The circuit equations can be solved analytically in the periodic case. The solution is expressed in the form of a dispersion relation which depends on the circuit parameters. The latter can be obtained by fitting to a corresponding dispersion relation obtained from a full 3D numerical solution. This calculation involves a single cell, with appropriate Floquet boundary conditions.

Procedure

- Solve in the frequency domain (2nd order equations).
- Loop equations for individual cells (I is unknown)
- Nodal equations for the manifold (V is unknown)
- Solve a deterministic system over a range of frequencies (2000 samples) (Spectral Function Method)
- From the value of I , compute V for each cell
- Using V , compute the sum wake
- invert the Spectral Function (sine transform)

Observations

- The sparsity structure of the system remains the same for each of the 2000 frequency steps.
- The solution at step $n-1$ may provide a good estimate for step n
- matrix is complex symmetric (reciprocity)
- When manifold voltage is not eliminated, matrix is block tridiagonal

Sparse Linear Equation Systems

- Standard LU factorization is $O(N^3)$.
- Storage Requirements $O(N^2)$.

The computational efficiency of sparse solvers is due in part to the fact that they eliminate vacuous operations. Note that these operations can in principle be avoided in a dense solver with a simple test. The BLAS package usually performs such tests.

Iterative Methods for Sparse Linear Equation Systems

- quadratic form involving the residual $Ax - b$.
- minimize the quadratic form using a Krylov method
- To improve efficiency, precondition (transform) the system prior to minimization step. Note: the preconditioning matrix should be sparse. Incomplete Factorization is a popular choice, but by no means the only possible one.

Direct Methods for Sparse Linear Equation Systems

- Based on Gaussian elimination
- Standard (LU) factorization favors maximum numerical stability
- With a different ordering of the elimination, one can minimize fill-in and greatly improve efficiency
- Many strategies available to find a more optimal ordering. Simple strategy: minimize fill-in at every step.

Iterative vs Direct Solvers

- Tests with various iterative methods were performed. Results with QMR and incomplete factorization were somewhat disappointing, comparable to LAPACK/BLAS. Efficiency greatly depends on the preconditioner. The solution for previous step can be used as a starting point. It may also be possible to reuse the preconditioning matrix for a few frequency steps.
- Tests with a direct sparse solver based on an optimized ordering strategy proved more encouraging. Although finding the optimal ordering is computationally expensive, **this needs to be done only once.**
- Although we have not completely given up on iterative solvers, the performance provided by an optimized ordering direct solver proved to be satisfactory for our needs.

Results !

- RDDS1 Matrix, manifold voltage not explicitly eliminated. $N = 618$
- no explicit inverse used to obtain the cell voltage
- Pentium III 550MHz GNU compiler
- Total Cpu time: 12.05 seconds
- Matrix assembly represents approximately 60% of this figure.

Loose Ends

- Boundary Conditions
 - “open”: $a_0 = a_1$, $\hat{a}_0 = -\hat{a}_1$ (currently in use)
 - “shorted”: $a_0 = a_2$, $\hat{a}_0 = 0$
- Manifold below cutoff frequency
 - The manifold section phase advance involves a cutoff term of the form $\sqrt{(1 - F_c/f)}$ Attention must be paid to the interpretation of this term when $f < f_c$. f varies from 14.0 to 16.5 GHz. F_c is slightly larger than 14 GHz in the last few cells.