Modeling of Mismatch and Halo

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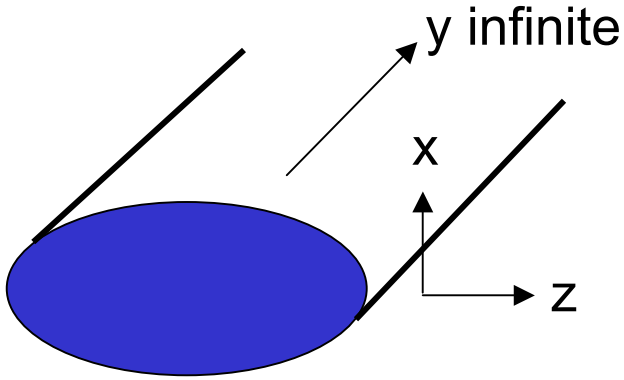
Questions

1. What determines rms emittance growth?
2. Is the 2:1 parametric halo really the only one of concern?
   - Other resonances (higher order) exist
   - What determines maximum halo radius?
3. Need to discuss non-symmetric (anisotropic) beams -> bridge gap to linac design!
4. Explore dependence on k_z/k_x similar to equipartitioning (does mismatch have similar resonant behaviour?)

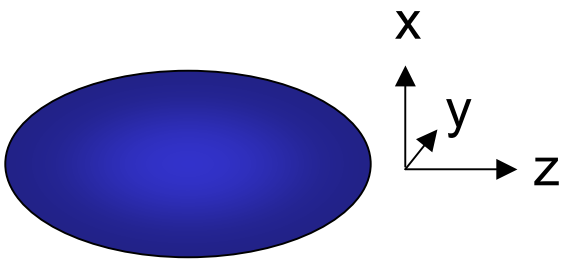
Anisotropy effects:
Ikegami (KEK) (NIM 2000)
Fedotov, Gluckstern et al. (PR 1998/99)

Franchetti, Hofmann (subm. PRL 2002)

2D modeling of 3D problem



2D: KV-theory + PIC

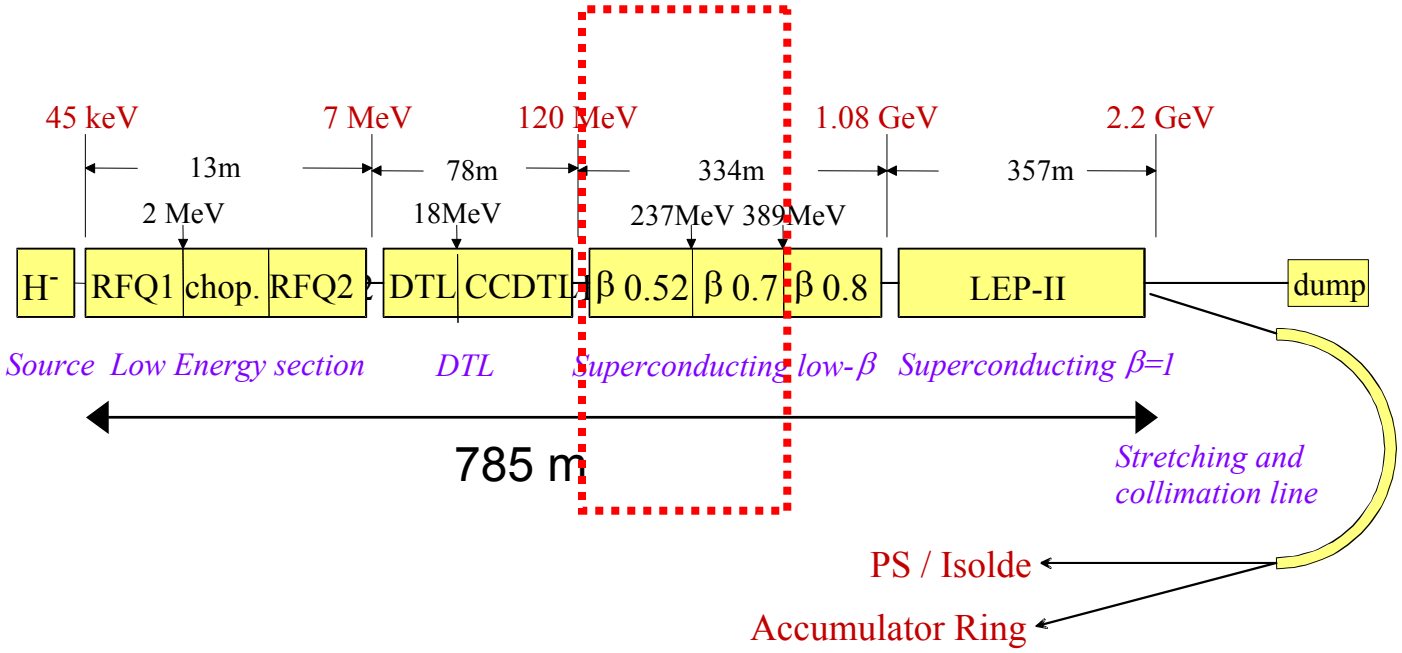


3D: PIC-simulations

- 2D PIC much faster to scan large parameter space
- encouraged by non-equipartition studies where 2D – 3D comparison was very good

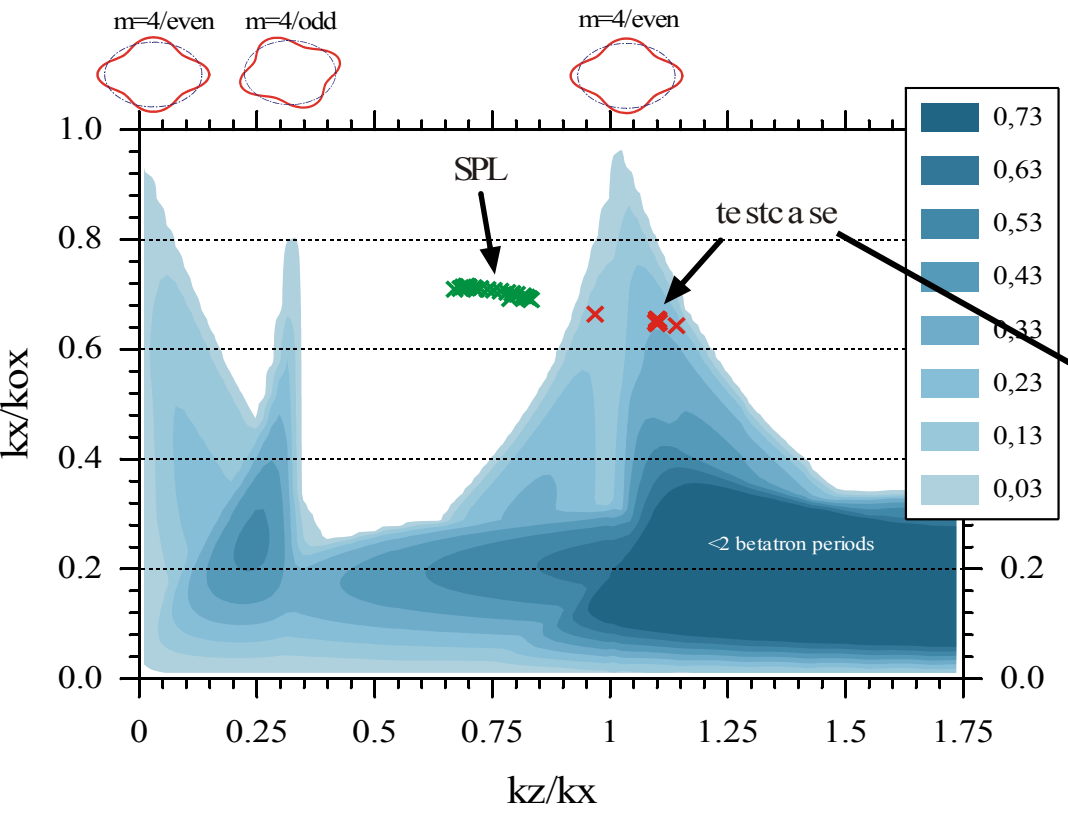
CERN SPL Linac Study: Non-equipartitioned

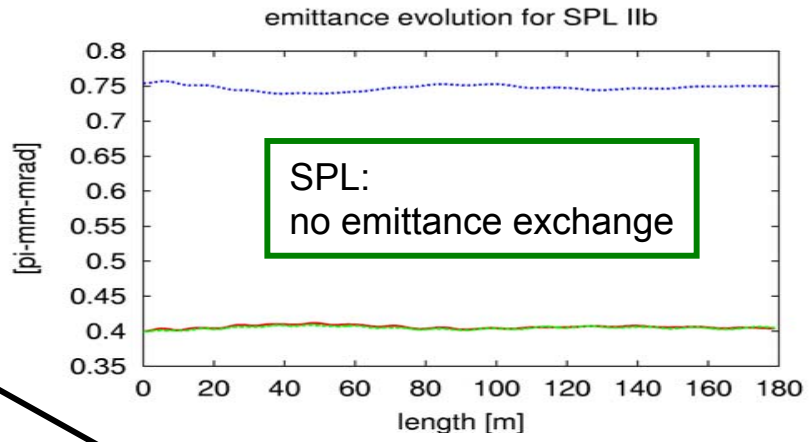
F. Gerigk/CERN

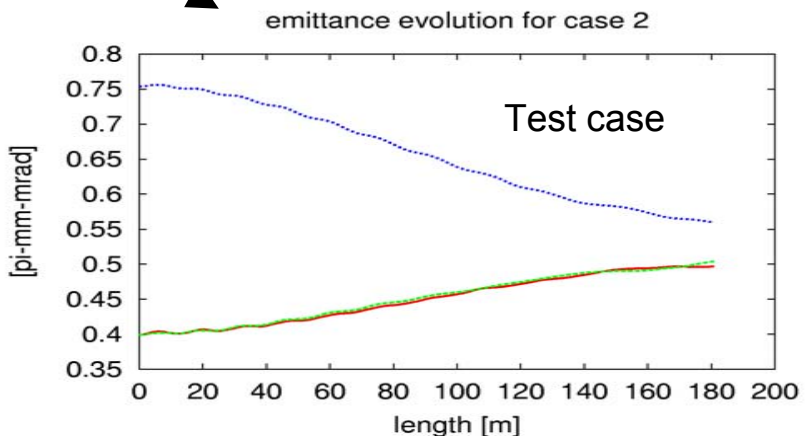


CERN SPL study: $\varepsilon_z/\varepsilon_{x,y}=2$

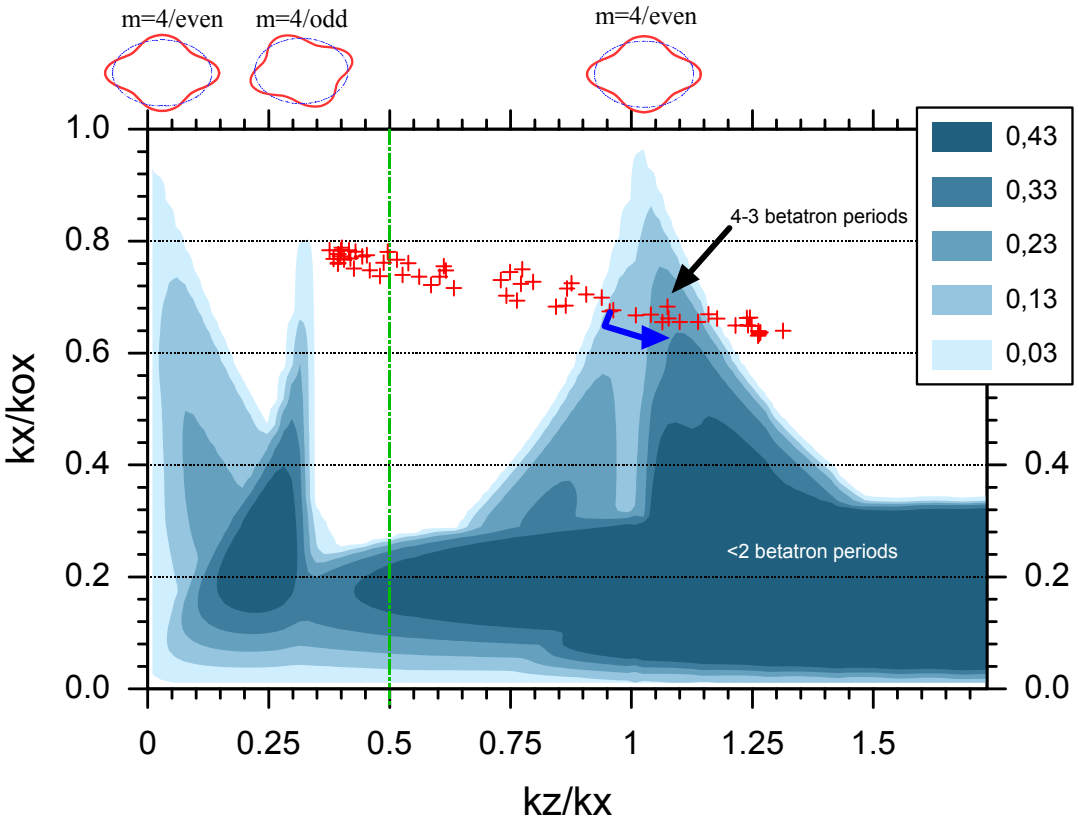
F. Gerigk, CERN

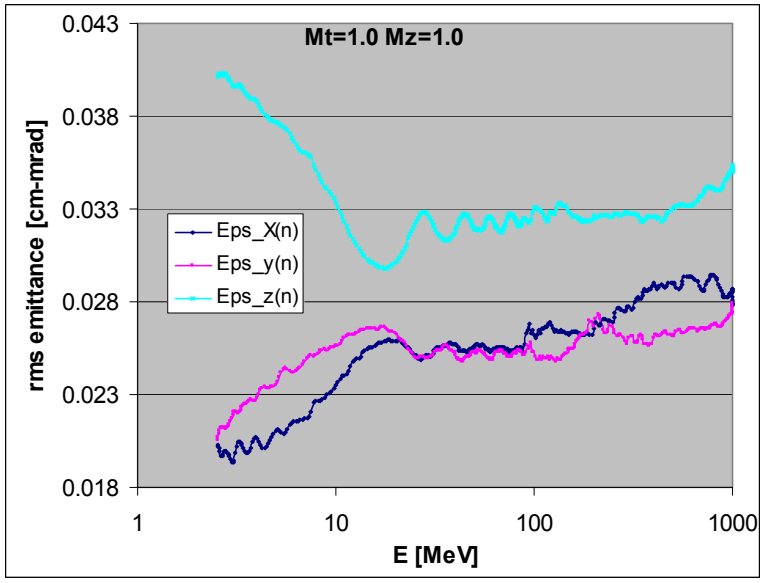






SNS Linac: assume $\varepsilon_z/\varepsilon_{x,y}=2$ (nominal 1.4)





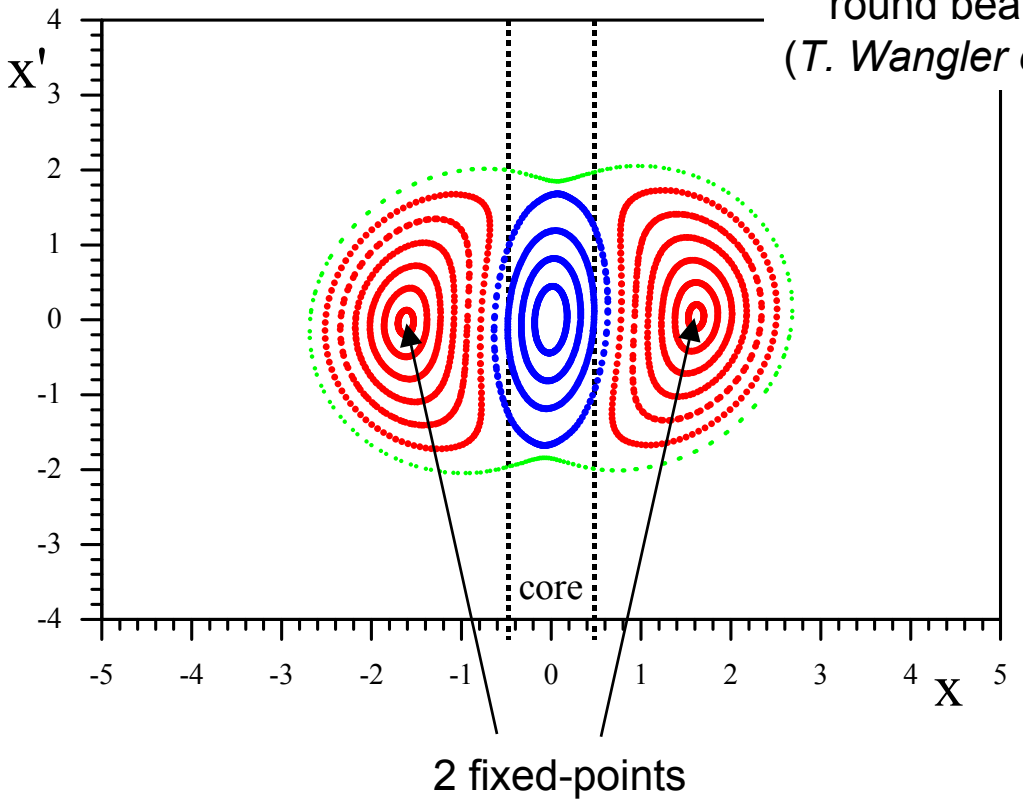
Parmila (D. Jeon, ORNL):

„Inevitable“ transverse emittance increase due to 2:2 ($2k_z/2k_x\sim1$) resonance

Parametric halo 2ν=1 resonance (2:1)

- Halo „absorbs“ mismatch energy => halo growth
- „most“ work on breathing mode for axi-symmetric beams

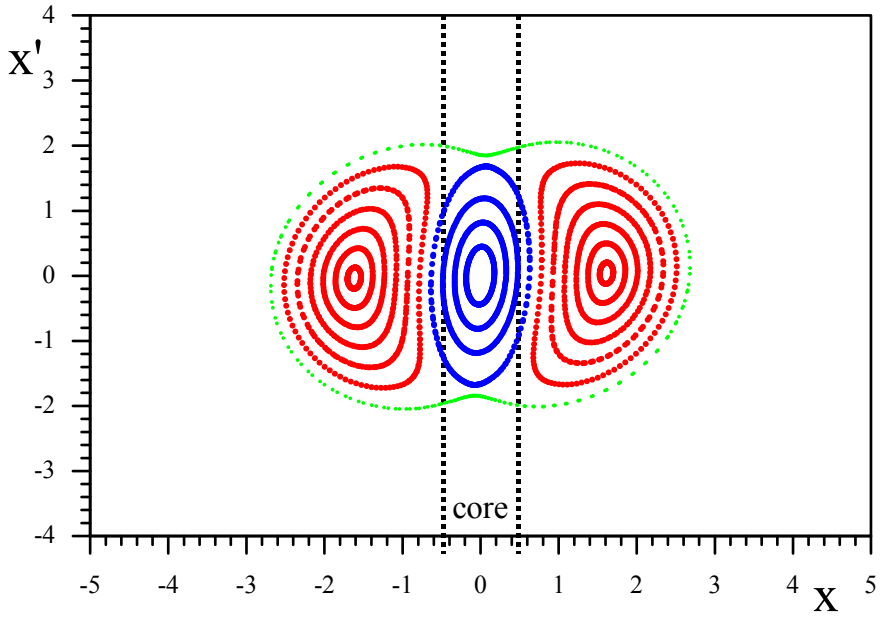
round beam „breathing mode“
(*T. Wangler et al., PRST-AB 2000*)



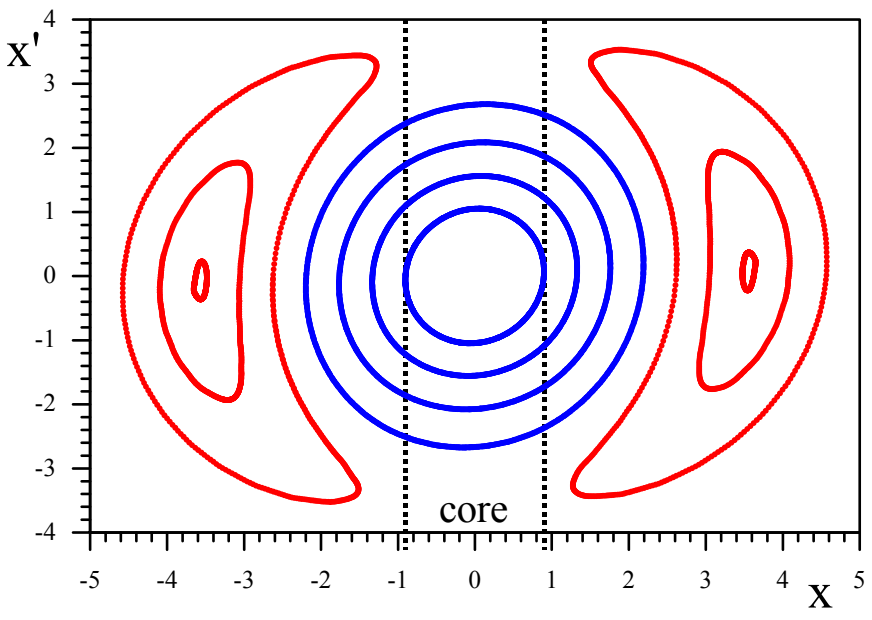
- in linac anisotropy – **new phenomena arise**
- we model them by using 2D PIC simulation

Maximum halo radius estimates for round (in 3D spherical) beams do not apply

Core-test-particle calculation for anisotropic beams

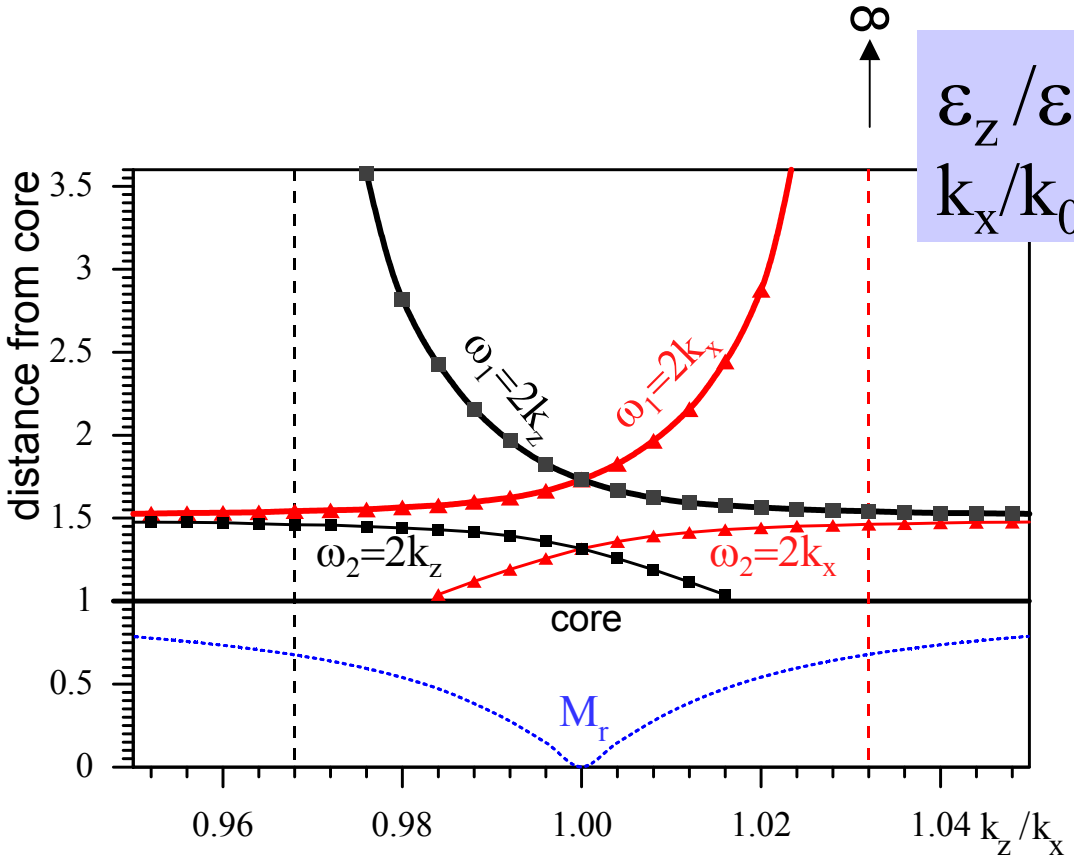


Round beam breathing mode



Weaker focusing in x ($k_x < k_z$)
- may not be easy to populate!

Interpretation: key is 2:1 parametric resonance fixed-point position relative to core



$\varepsilon_z/\varepsilon_x=1$

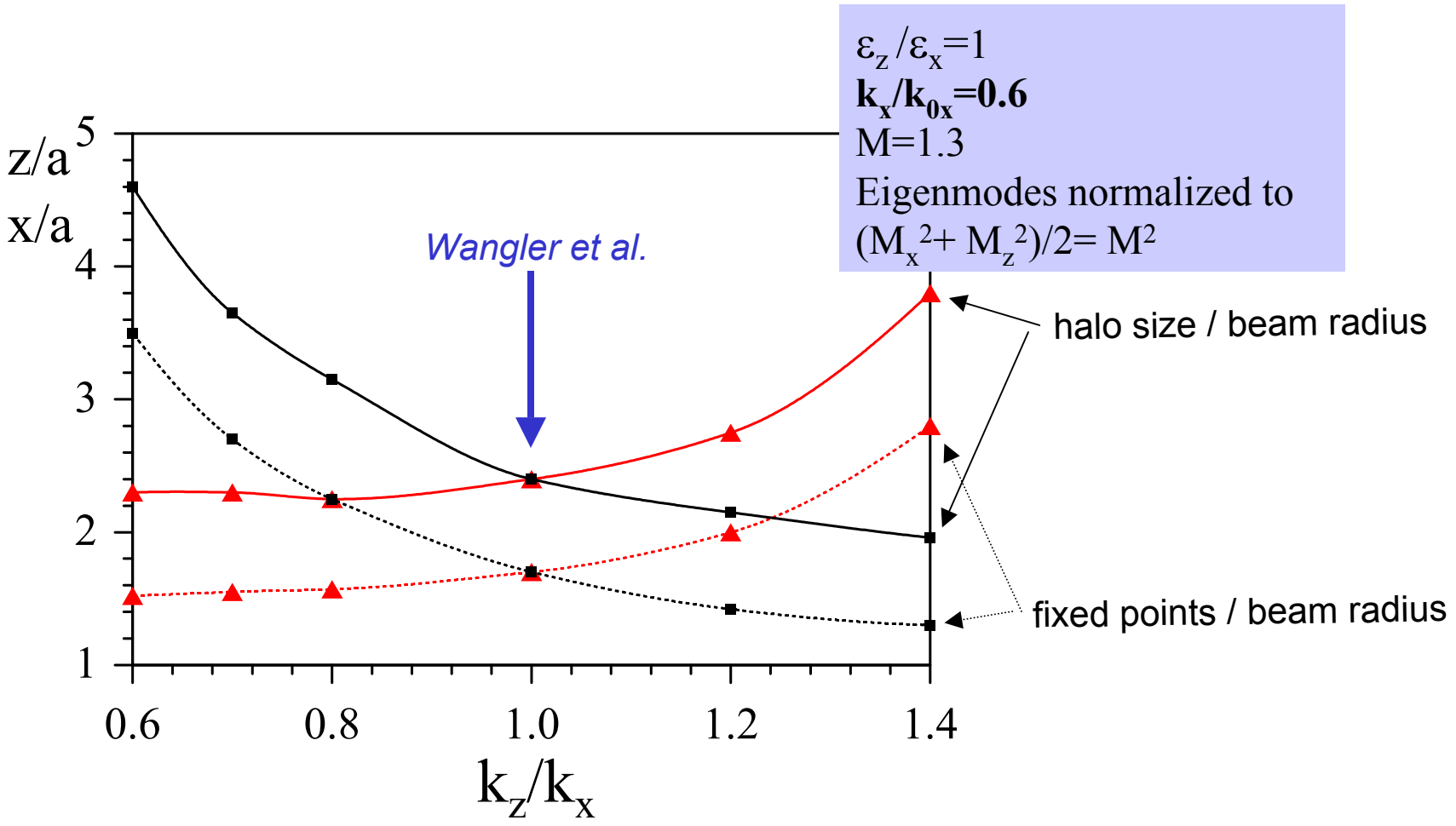
$k_x/k_{0x}=0.95$

Fixed-points in x-plane

- move away from core for slow and fast eigenmode if x-focusing weaker ($k_z/k_x > 1$)
- move closer if x-focusing stronger
- likewise maximum halo radius

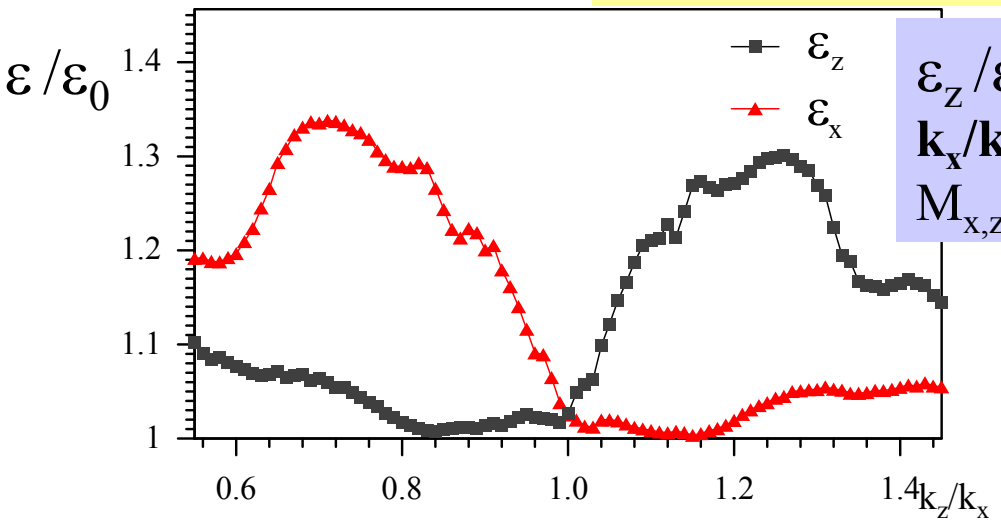
parametric halo „attracted“ / “repelled“ by stronger / weaker focusing

Confirmed by core-test-particle simulation for anisotropic beams



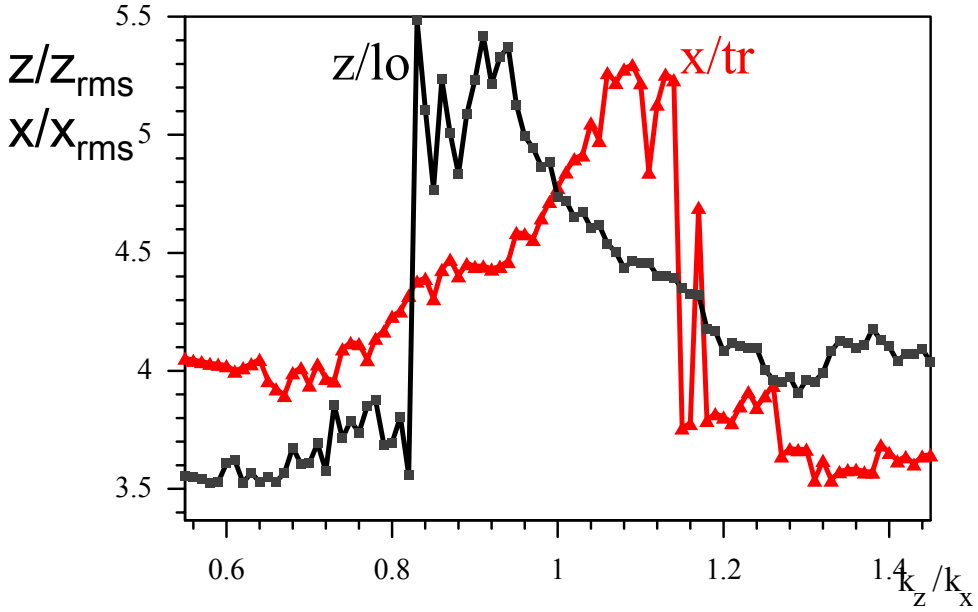
Rms emittance growth larger for stronger focusing direction = closer fixed-points

waterbag beam PIC simulation



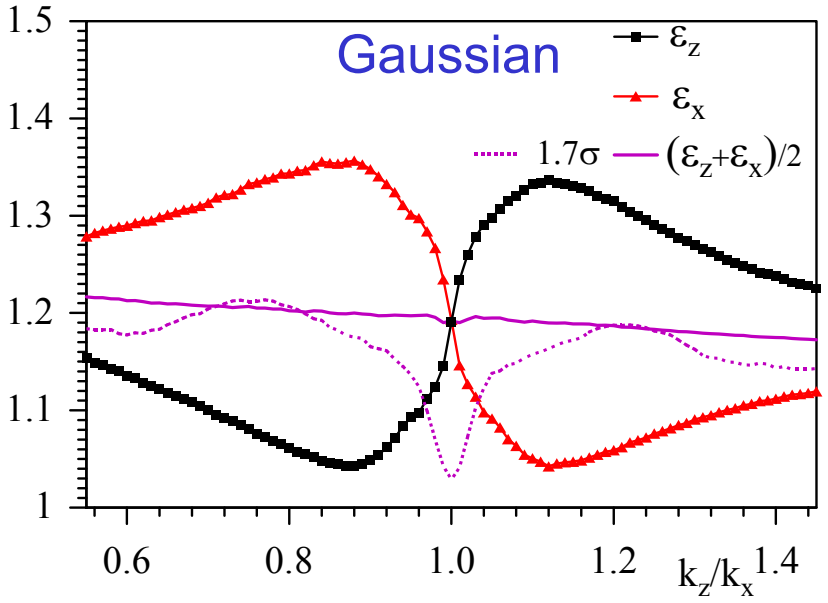
$\varepsilon_z/\varepsilon_x=1$
$\mathbf{k_x/k_{0x}=0.6}$
$M_{x,z}=1.3$

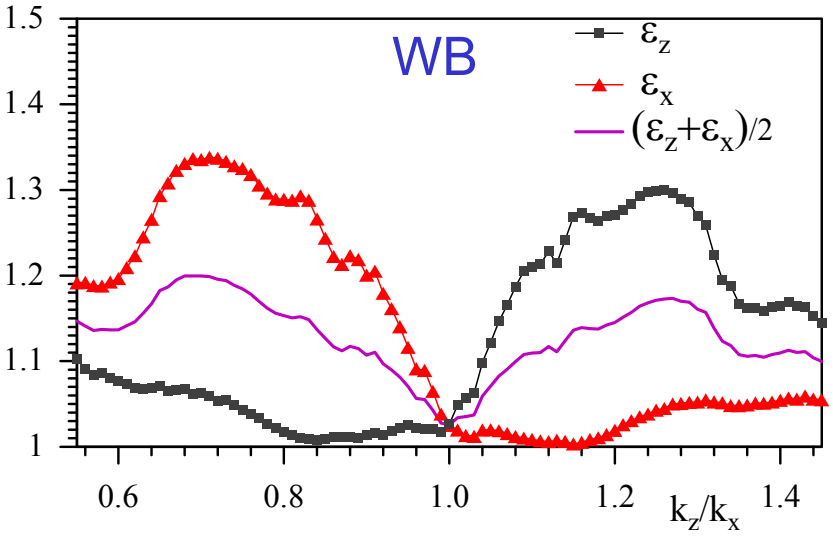
Anisotropic beam results from isotropic mismatch
„hotter“ direction heated!
-> further away from EP



- Halo growth opposite to rms emittance growth
- No transport to resonance if fixed-point too far!
- Transverse halo larger for $k_z/k_x>1$

Gaussian (initial tails from source/RFQ)





$\varepsilon_z/\varepsilon_x=1 \quad k_x/k_{0x}=0.6$

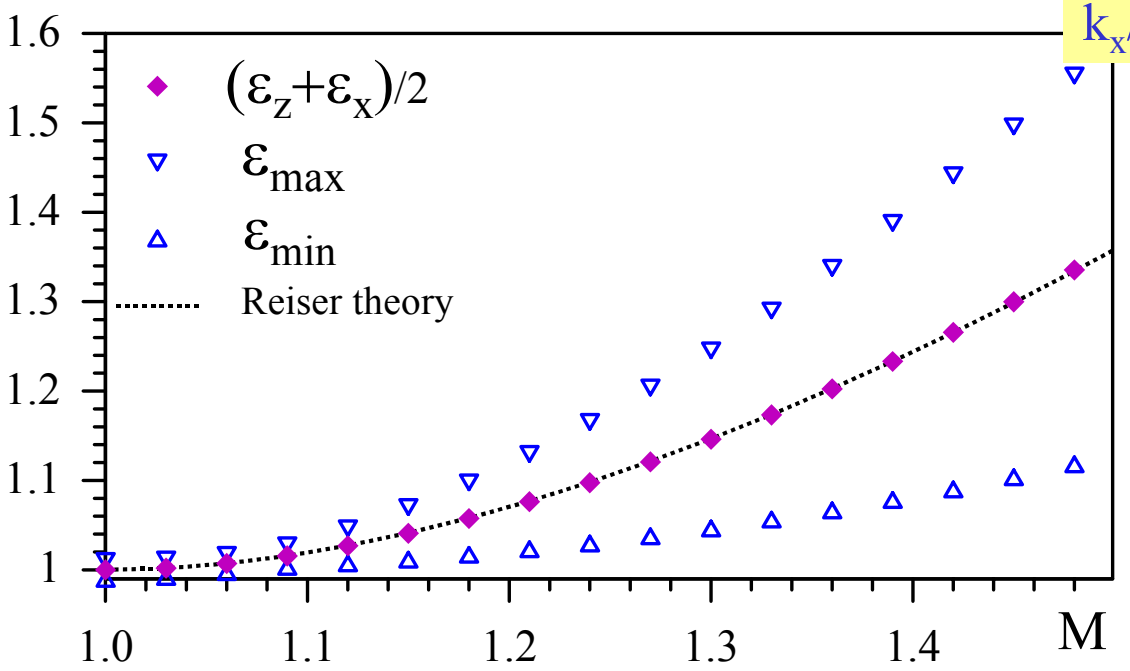
Gaussian:

- Smooth response
- Find $\varepsilon_z+\varepsilon_x$ ~ constant for full G
- Nearly full conversion of free energy (*Reiser et al., 1991)*
- Energy truncated (1.7σ) -> WB

Suggest for experiment:

- Even symmetric focusing yields full conversion of field energy if enough tails
- Split transverse focusing
- Anisotropy of rms response strong signature for all distributions

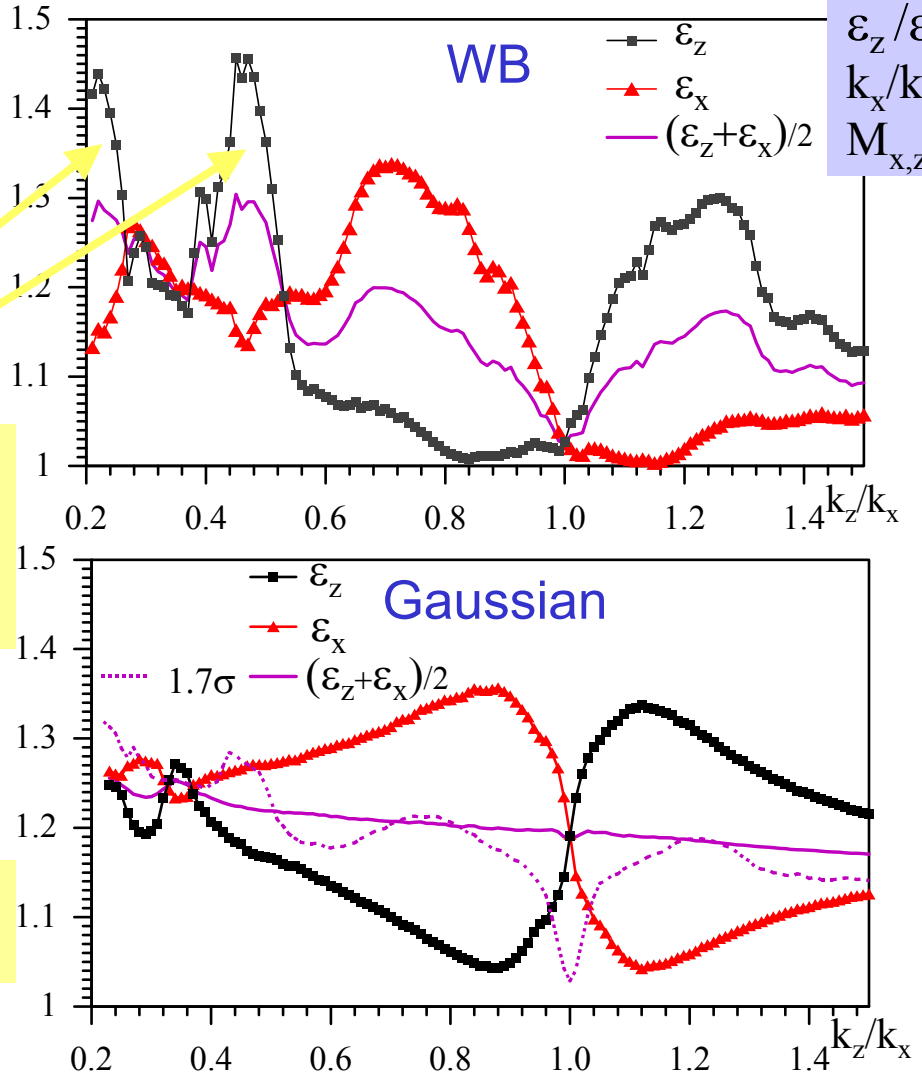
Free energy conversion not isotropic (Reiser, 1991) but directional - into stronger focusing direction



weak space charge:
$k_x/k_{0x}=0.95$

Cucchetti, Reiser, Wangler, 1991
(round beam breathing mode, Gaussian):
Mismatch free energy conversion:
~ 100% into rms emittance growth for weak space charge,
~ 80-90% if stronger space charge

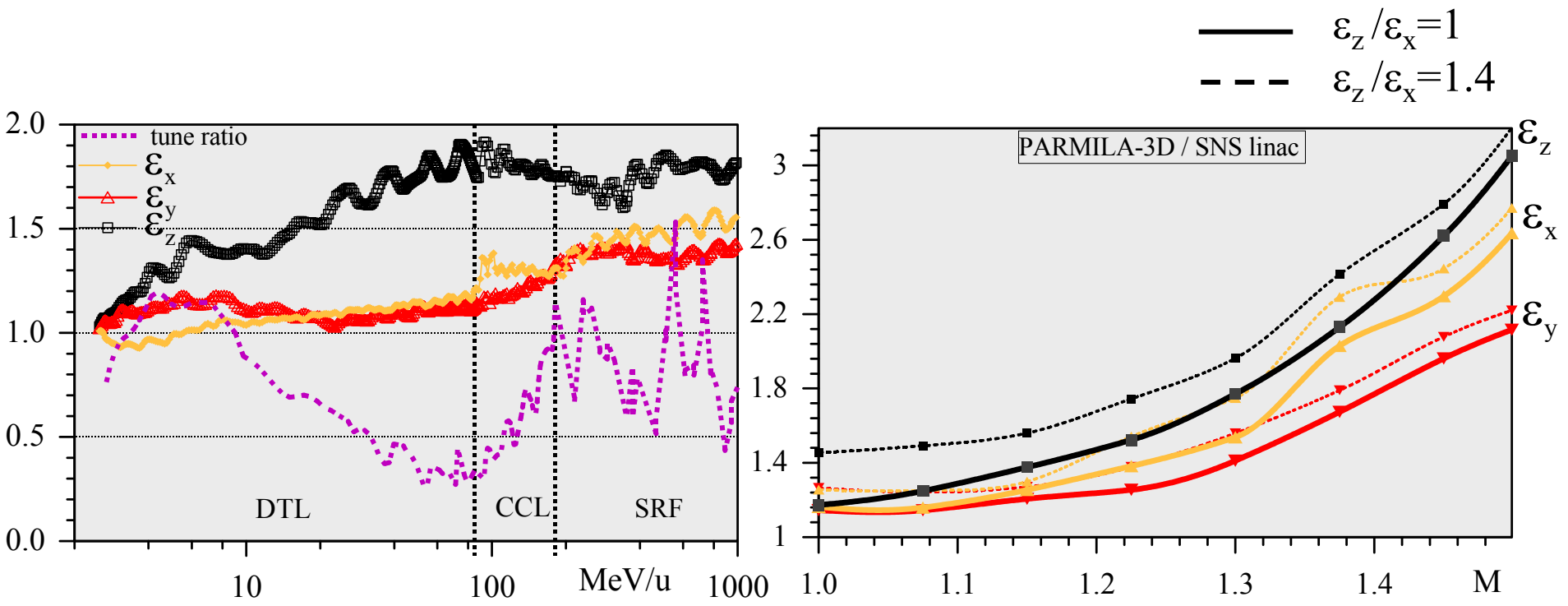
Stronger space charge k_x/k_{0x}=0.6 and larger range of k_z/k_x parametric halo not the only one!



Indicates higher than 2:1 resonance suggest 4th order: $2k_z-2k_x=\omega_{lo}$ or $4k_z=\omega_{lo}$

Largely disappears for Gaussian

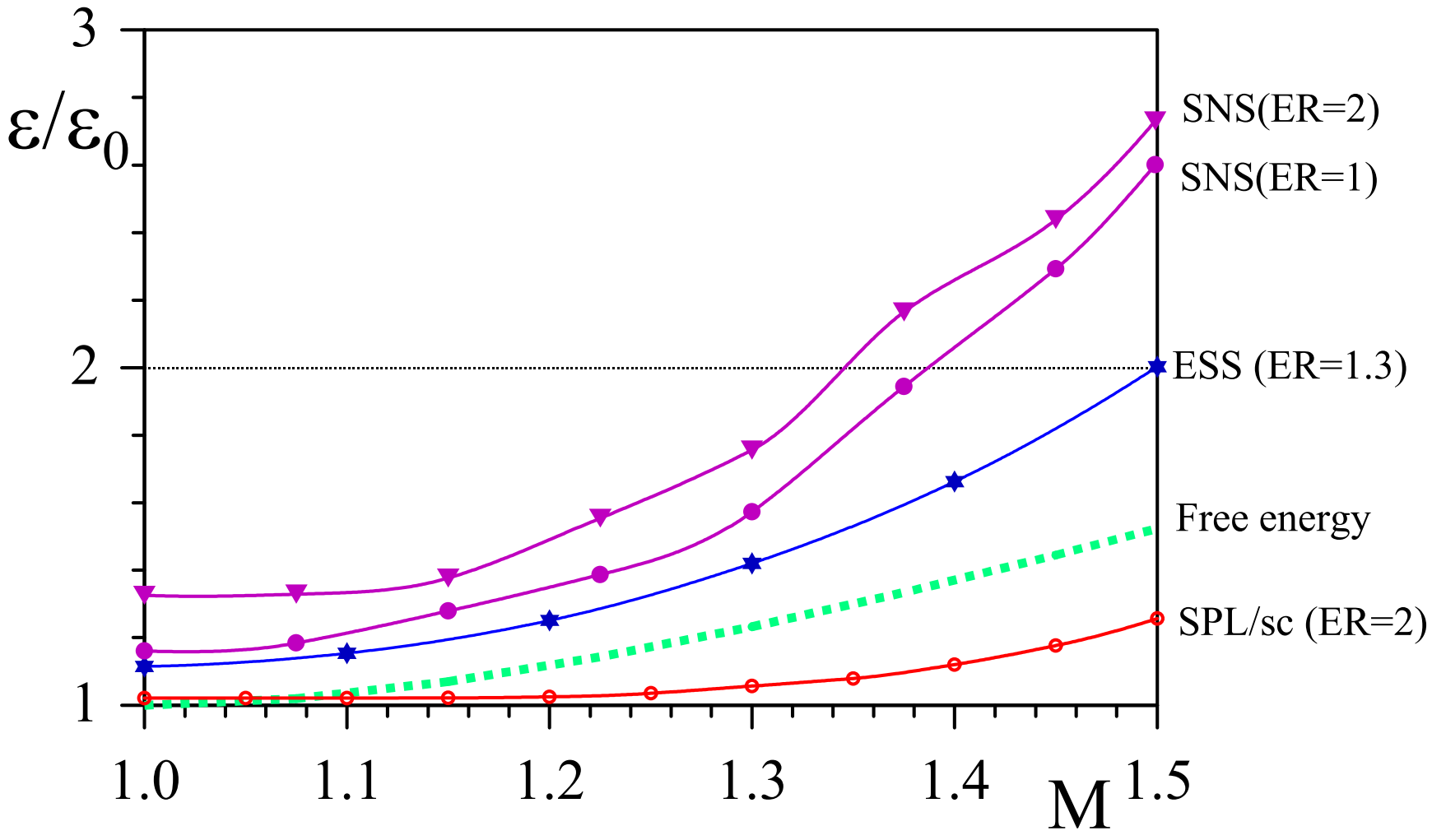
Application to SNS Linac



Dropping into region k_z/k_x<0.5 might explain enhanced longitudinal growth

Comparison of $(\varepsilon_z + \varepsilon_x + \varepsilon_y)/3$ with "free energy limit"

assuming full conversion (symmetric mismatch in x,y,z)



Conclusion

- Explored conversion of mismatch into *anisotropic* rms emittance growth
  - *full free energy conversion for symmetric focusing if tails in distribution*
  - *anisotropy caused by attracted or repelled fixed-points of 2:1 resonance (anti-thermodynamic behaviour)*
  - *mismatch „energy“ $M^2=(M_x^2+ M_y^2+ M_z^2)/3$ more practicable than eigenmode distinction*
  - *transverse halo size can somewhat exceed round / spherical beam case if $k_z/k_x>1$*
- Found that parametric halo not the only one for $k_z/k_x<0.5$, but weak effect on halo size
- Linacs may deviate from theoretical free energy limit
- Next: will random errors wash out resonant behaviour?

halo build-up times (isotropic beams)

for symmetric focusing and AG-focusing