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Multiparticle Dynamics in the E-ϕ Tracking Code ESME

- Introducing ESME
- Space charge, impedances, and wakefields
- Recent refinements
  - time or (equivalently) bandwidth scaling
  - micro-matching of initial distribution
- Numerical evaluation of Jacobian
- Provocative results
  - coasting beam and high-Q passive resonator
  - negative mass instability

Meet ESME

Multiparticle tracking has established utility for modeling evolution of longitudinal phase space distributions of particles in accelerators as they respond to the rf in acceleration or bunch manipulation. ESME has been primarily developed for design studies of machine cycles and rf gymnastics using single particle equations of motion.

One goes from single particle to multiparticle dynamics by calculating the beam current every time step and including its effect on the single particle motion. However, the number of macroparticles needed and bandwidth required for quantities in the frequency domain need careful attention. It is very easy to generate spectacular spurious instabilities by long time steps or too few macroparticles. Considerable attention has been given in recent years to these considerations and to development of facilities for multiparticle dynamics.

Space Charge Model

The self-impedance from the direct interparticle forces is derived from an electrostatic calculation in the beam rest frame transformed to the lab frame.[1] The force arises from the gradient of the azimuthal charge distribution $\Lambda_n(\Theta)$, which can be evaluated from the fourier series for Λ for the frequency domain or directly for time domain. The impedance representing the direct particle-particle force is

$$\frac{Z_m^{sc}}{m} = -i\frac{Z_0 g}{2\beta\gamma^2}\,, \tag{1}$$

where $Z_0 = \sqrt{\mu_0/\varepsilon_0} = 377\Omega$ and g is a factor containing the dependence on beam and vacuum chamber transverse dimensions. For a uniform cylindrical beam of radius a centered in a smooth beampipe of radius b

$$g = 1 + 2\log(\frac{b}{a})\,. \tag{2}$$

In ESME this factor is scaled with beam momentum to account for the change in beam radius and rolled off at very high frequency to approximate the exact solution at high mode numbers. The impedance is pure imaginary, like an energy-dependent negative inductance; the bunch as a whole can not gain or lose energy through internal forces.

Frequency Domain Facilities

ESME is fundamentally time domain, but beam current and image currents can be fourier analized at harmonics of synchronous circulation frequency $\Omega_{s,n}$. At the end of the n-th turn the beam has a distribution of particles in azimuth and total energy $\Psi_n(E,\Theta)$, normalized to N, the total number of particles in the beam. For small spread in particle velocity, the distribution circulates some few turns without change of form. Then the current distribution is just

$$I_{b,n}(\Theta) = \frac{\Omega_{n,s}e}{2\pi}\int \Psi_n(E,\Theta)\,dE = \frac{\Omega_{n,s}e}{2\pi}\Lambda_n(\Theta)\ . \quad (3)$$

The distribution $\Lambda_n(\Theta)$ circulates with negligible change of shape if

$$\frac{\sigma_\tau}{\tau} = \frac{\eta\sigma_E}{\beta^2 E_s} \ll 1\ , \quad (4)$$

where the σ's are the rms spreads resulting from the energy variance

$$\sigma_E^2 = \iint (E - E_s)^2 \Psi(E,\Theta)\,dE\,d\Theta\ . \quad (5)$$

Generally the width of the energy distribution will be severely limited by available aperture.

The mapping approach additionally requires what will be called a quasistatic current distribution $I_{b,n+1}(\Theta) \approx I_{b,n}(\Theta)$ implying $\Omega_{s,n} \approx \Omega_{s,n-1}$. If one wants to write the current at the gap as a function of time there is the usual change of sign between phase and azimuth:

$$I_b(\Omega_s t) = \frac{e\Omega_s}{2\pi} \sum_m \Lambda_m e^{i(-m\Omega_s t + \psi_m)} \; . \qquad (6)$$

The fourier series for the current has been written with real amplitudes multiplied by complex phase factors, that is, as a sum of phasors.

The current produces a beam-induced voltage through the total longitudinal coupling impedance $Z_{||}(\omega)$; this quantity evaluated at $m\Omega_s$ will be denoted as the phasor $Z_m e^{i\chi_m}$. The beam-induced voltage is applied to each particle at time t_n when the synchronous particle is at the gap; that voltage depends on the relative phase between the particle and the current. The synchronous particle has phase 0. Thus, the energy increment for the i-th particle on the n-th turn resulting from the voltage induced by the beam current is

$$eV^b_{i,n} = -\frac{Ne^2\Omega_s}{2\pi}\sum_m \Lambda_{m,n} Z_m e^{i(m\Theta_{i,n}+\psi_{m,n}+\chi_m)} \, . \tag{7}$$

One should not cringe at the absence of synchrotron sidebands in this expression. They are generated in the tracking the same way they are generated in a sychrotron — by the phase modulation.

Time Domain Facilities

1. Gradient of Λ calculated at each particle by cubic spline interpolation of over bin of particle and two immediate neighbors

2. Green's function solution for simple resonance(s) with turn-to-turn accumulation

3. Wakefield for arbitrary distribution and arbitrary $Z_{||}$ using response to triangular unit current pulse calculated by TCBI or like

4. Free inter-mixing of time domain and frequency domain

5. Transient fundamental beam loading with optional feedback and/or feedforward correction

Scaling Concept

It appears that the phase space motion can be accelerated by scaling the phase slip factor η and the potential up by the same factor, hereafter λ. The potential is not necessarily just that from the rf system; the collective potential enters identically.

- The particle distributions are identical when time t in the un-scaled calculation is compared to time t/λ in the scaled calculation.

- Obvious gain is a factor λ^{-1} in the computing time by speeding up the clock in the scaled calculation.

- Scaling up of the time means that frequencies like the rf frequency and resonance frequencies in $Z_{\|}$ must also be scaled.

- Space charge term in frequency domain

$$\frac{Z_{\mathrm{SC}}}{n} = -i\frac{Z_\circ g}{2\beta\gamma^2}$$

- Evaluation not compromised by more widely spaced frequency sampling

- Number of macroparticles can be scaled by λ^{-3} with same level of numerical noise

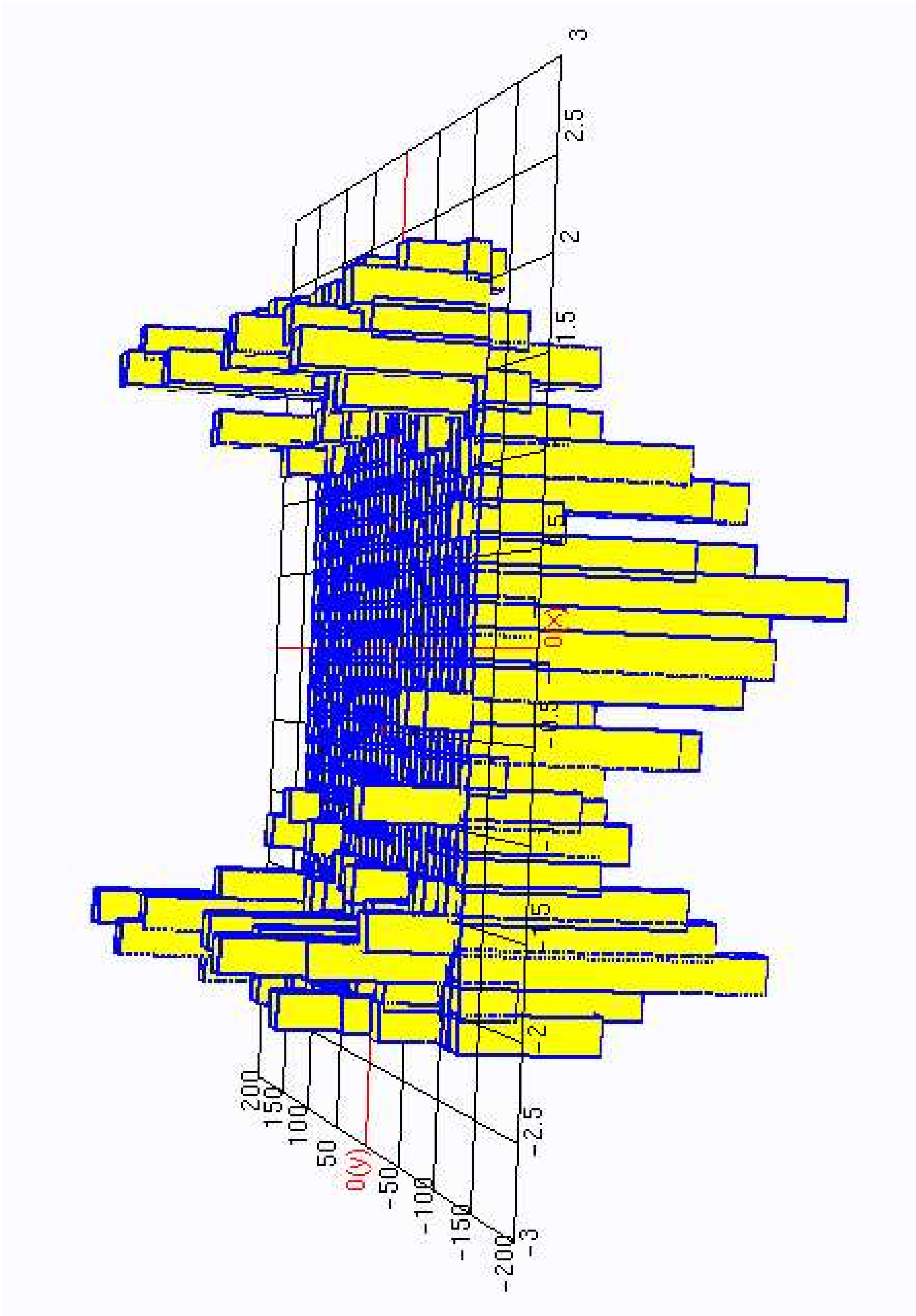
  - shown rigorously for space charge[3]

  - heuristic frequency domain argument for any smooth $Z_{||}(\omega)$[4]

- Possible gain from scaling thus λ^4 in many cases

Numerical Evaluation of Jacobian

The Jacobian can be evaluated directly from the numerical results of a single iteration of the map to establish what the effect of numerical precision is on the conservation of phase space area. The following figure shows $1 - J$ evaluated with differences of 0.02° and 0.02 MeV between macroparticles; it shows the minimum maximum magnitude obtained in a trade-off between improved accuracy in the derivatives and increased truncation error with a finer grid, *viz.*, $3 \cdot 10^{-12}$. Some systematic variation in $1 - J$ is apparent, partially obscured by numerical noise. It has the form $\cos\varphi$.



Numerical evaluation of 1 - Jacobian with cell size $0.02^\circ \times 0.02$ MeV. Max = $3 \cdot 10^{-12}$.

Micro-matched Initial Distributions

- Tracking often shows charge-dependent emittance growth well below expected instability limits because initial distribution is not an equilibrium solution to the multiparticle EOM — not matched

- Even for single particle potentials there are only a few easily used well-matched initial distributions.

- The most general approach to obtaining a matched distribution is to transform adiabatically from a matched case to the desired case, e.g., by turning on beam charge slowly or by capturing slowly from a coasting beam.

- The elliptical distribution is matched at zero charge; it makes a reasonable starting point.

Application to Coasting Beam Self-Trapping

- Excitation of a passive resonator by coasting beam and beam response to the resonant voltage is an archetypal multiparticle dynamics problem.

- One expects that beam will self bunch above threshold, with the bunches decelerated from the initial beam energy.

- This example illustrates that for R/Q sufficiently high, bunching is dependent on relation of resonant frequency to nearest harmonic of beam circulation frequency.

- In either case mean energy is decelerated equally, either in buckets or by phase displacement.

- animations with parmeters like PSR plus h=3 resonator animated GIF files viewable with a browser or viewer like JPEGVIEW (files psr3hi.gif and psr3lo.gif)

- The relation to Robinson instability (h=1) is suggestive.

Application to Negative Mass Instability

- Attempts to compare the Hardt analysis of negative mass instability [*cf.* also Neil & Sessler] to numerical tracking have run up against particle statistics problems deriving from computer speed and memory limitations.

- It is timely to try again because
  - now even ordinary computers have GHz clocks and Gbyte memories
  - scaling derived from Vlasov equation appears to ameliorate both problems
  - numerically quiet distributions of a reasonable number of macroparticles can be (at least initially) even quieter than the real beam distribution

Qualitative Results

- The maximum growth rate occurs near 60 GHz regardless of time/bandwidth scaling.

- The frequency of fastest growth is much less than predicted by Hardt for the parameters used. ($\sim 0.5\times$)

- The amount of emittance growth appears comparable but systematically less with larger scaling factor.

- Computers are now fast enough and big enough that a calculation could be done for a physically interesting case with the actual number of particles in a bunch, thereby eliminating any question about scaling or numerical noise.

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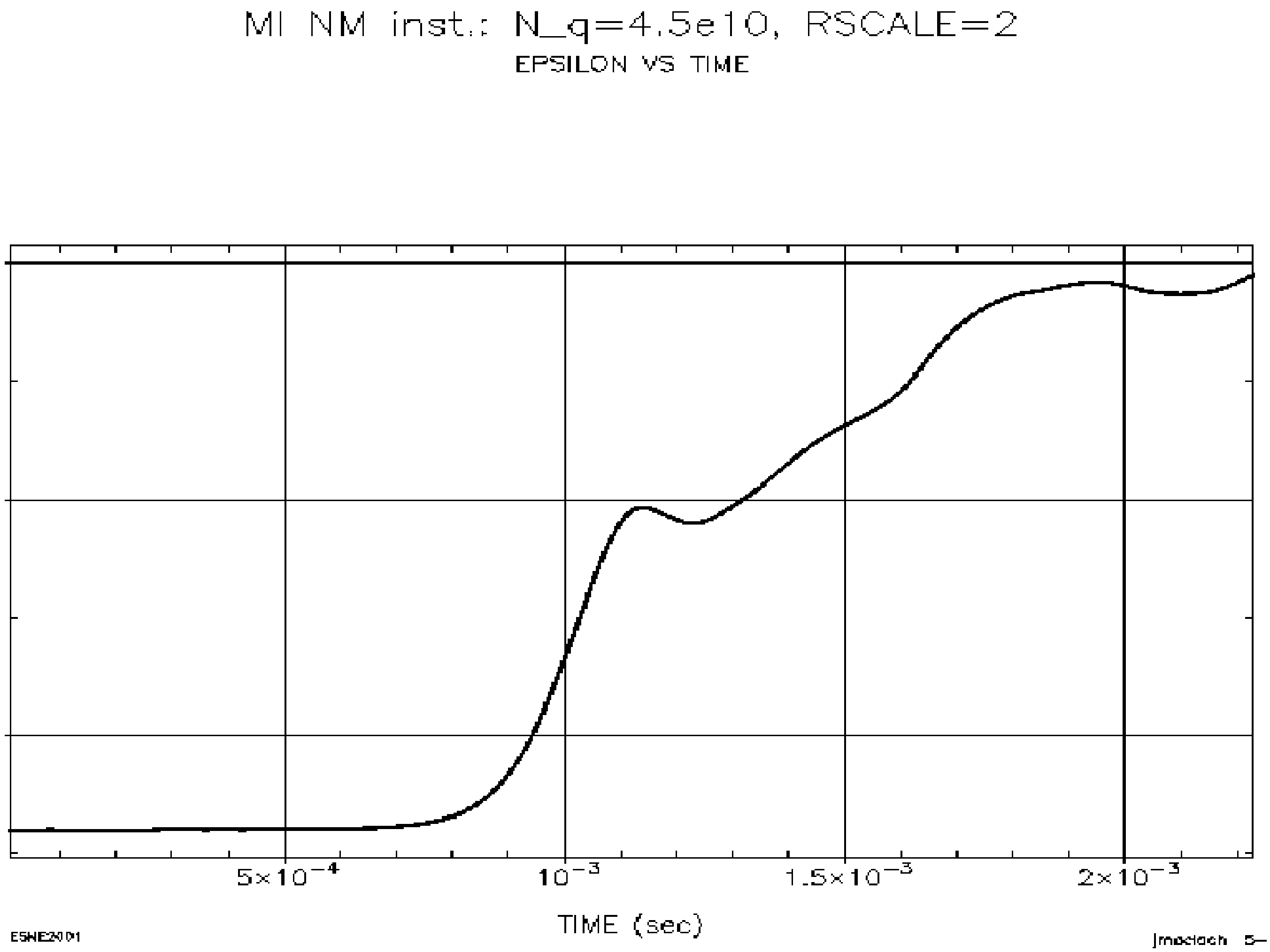
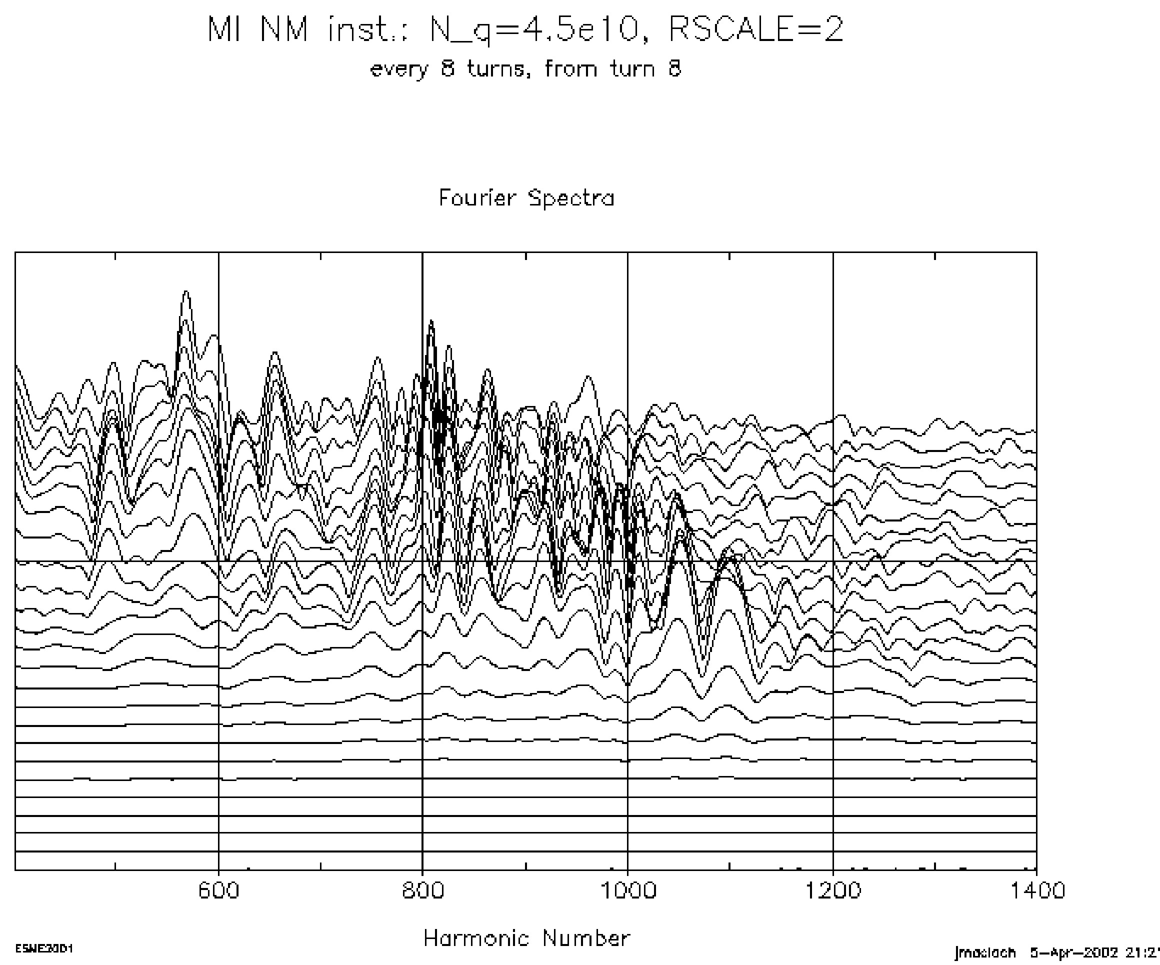
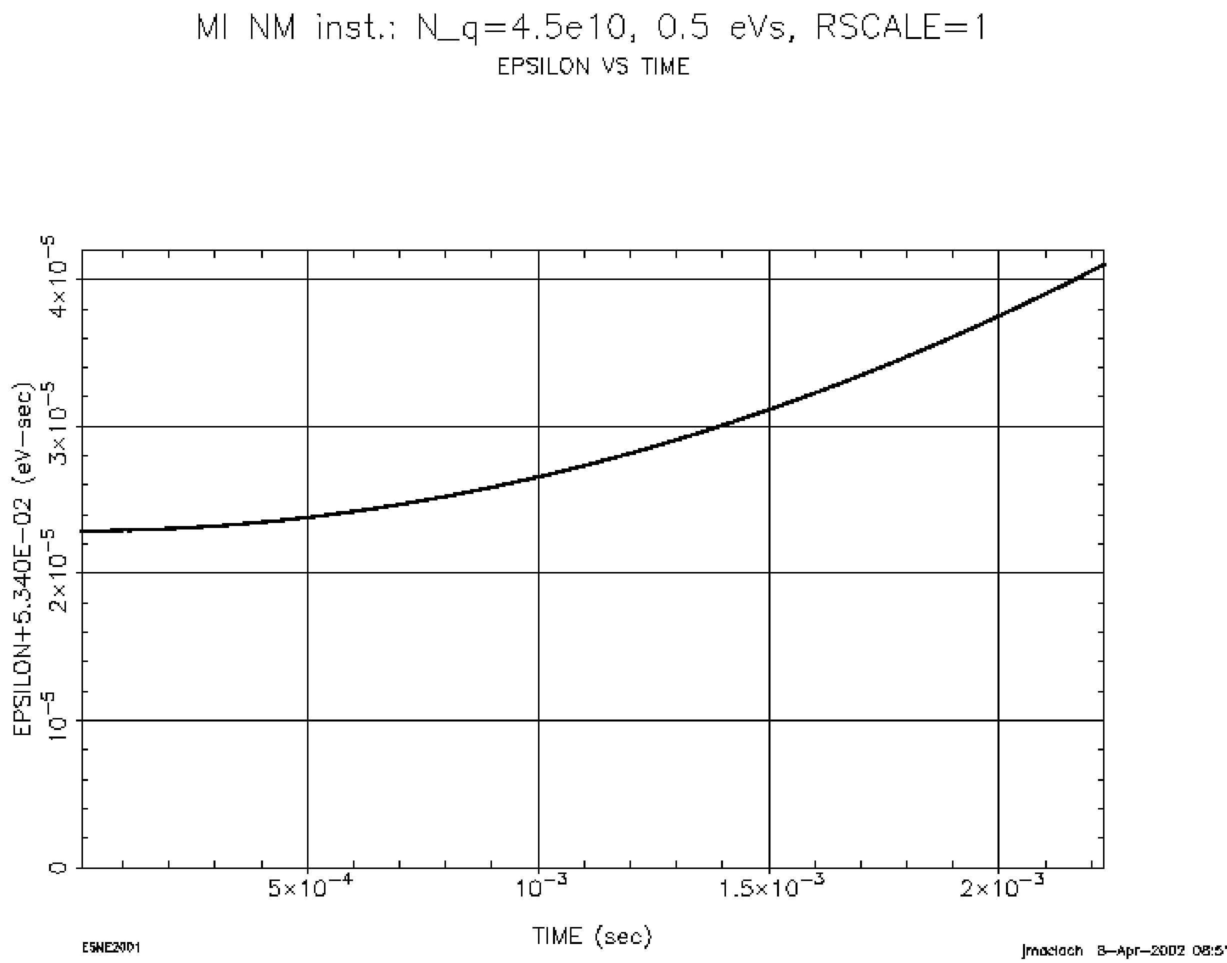


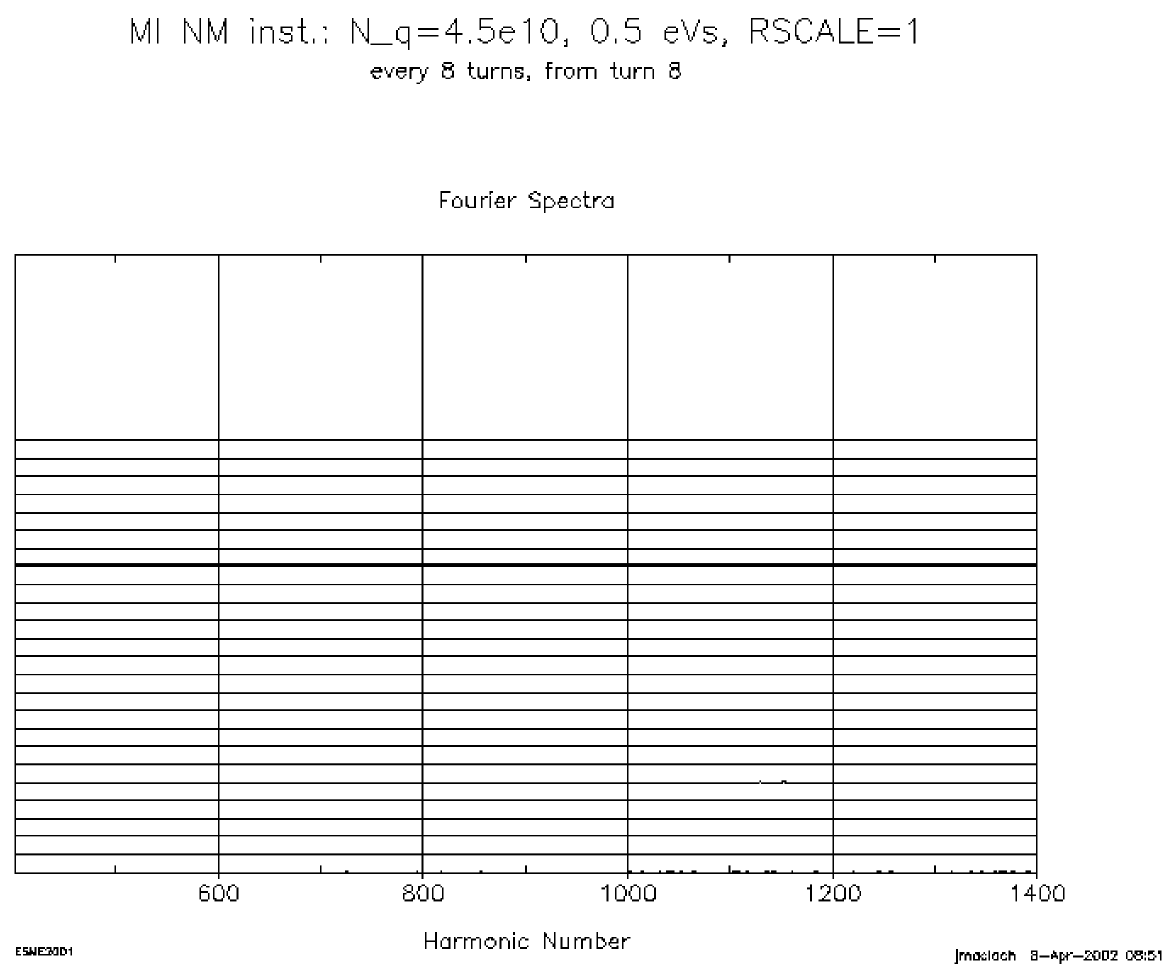
Emittance *vs*. time for 0.2 eVs bunch of $4.5 \cdot 10^{10}$ protons just above transition ($\eta = 8.9 \cdot 10^{-5}$), other parametrs from the Fermilab Main Injector



Fourier spectrum at times separated by 88 μs for bunch described in preceding figure. The abcissa is labeled in harmonics of the 53 MHz rf; the frequency span shown is 21 – 74 GHz.



Emittance *vs*. time for a 0.5 eVs bunch with parameters otherwise the same as in the previous two figures



The Fourier spectrum at 88 μs intervals for the 0.5 eVs bunch on same scale used for 0.2 eVs bunch

Parting Thoughts

- Numerical modeling in dynamics is especially helpful to
  - look for gross oversights
  - carry practical calculations beyond threshold
  - evaluate result of simultaneous action of several processes

- Graphical output from macroparticle models can communicate results easily to non-experts.

- Reasonably well-tested codes are a useful source of comparisons or benchmarks.

Some References

1. J. A. MacLachlan, “Longitudinal Phasespace Tracking with Spacecharge and Wall Coupling Impedance”, Fermilab FN-446 (2/87)

2. J. A. MacLachlan and Z. Nazario, “Scaling for Faster Macroparticle Simulation of Longitudinal Dynamics in Synchrotrons and Storage Rings”, PRST-AB **4** 019001 (2001)

3. J. Wei, “Longitudinal Dynamics of the Non-Adiabatic Regime on Alternating Gradient Synchrotrons”, PhD dissertation for State University of New York at Stony Brook (1990)

4. J. A. MacLachlan, “Particle Tracking in E-φ Space for Synchrotron Design and Diagnosis”, FERMILAB-Conf-92/333 (1992)

5. J. MacLachlan, “Difference Equations for Longitudinal Motion in a Synchrotron”, Fermilab note FN-529 (1989)

6. Etienne Forest, “Canonical Integrators as Tracking Codes (or How to Integrate Perturbation Theory with Tracking)”, SSC-138 (September 1987), unpublished

7. The current documentation, containing references to the underlying principles, is most accessible on the ESME web page `www-ap.fnal.gov/ESME`