Space Charge Resonances in High Intensity Drivers

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Outline

- Coherent Space Charge Tune Shift
  - Analytical Approach
  - Coupled Mode Coefficient
- Lattice Driven Resonances (2D)
  - Half-Integer Resonance (SNS)
- Space Charge Driven Resonances

Require systematic approach

1. For the GSI project of a large synchrotron SIS100 as driver for radioactive ions and antiprotons we need to keep loss <1% (~1 sec) - main problem seems wall desorption (10^4 ions per incident heavy ion)

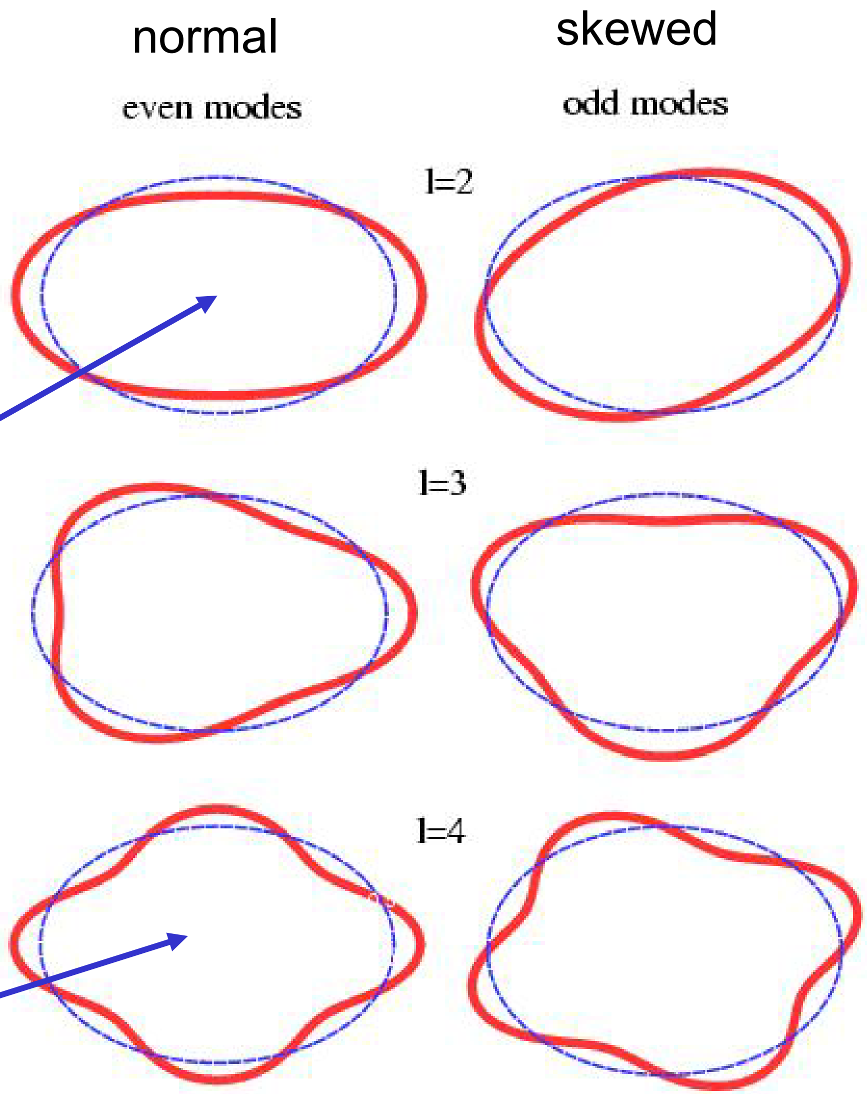
1. 3 kinds of space charge effects:
   - tune spread/shift -> „frozen-in“ space charge for up to ~10^5 turns (talk by G. Franchetti)
   - sc as source of nonlinearity
   - coherent dynamical effect -> self-consistent space charge calculation for ~10^3 turns - compare with theory (this talk); applies to coasting beam or ideal barrier bucket

Hofmann, Beckert, 1984
Machida, 1991 ->
Baartman, 1998
Venturini, Gluckstern, 2000
Burov, 2001
Fedotov, Hofmann, 2002

Transverse Space Charge Modes

Eigen modes of transverse density oscillations

Coherent motion induces additional shift
-> needs selfconsistent (kinetic) calculation



2 envelope modes
(breathing and quadrupolar)

Normal octupole or
Space charge of WB beam

Derived dispersion relation from Vlasov-Poisson equation for l = 2, 3, 4

*Hofmann., Phys. Rev. E **57** (1998)*

The coherent frequency $\sigma=\omega/\nu_x$ must be solved in 3-dimensional parameter space:

$\alpha=\nu_y/\nu_x$, $\eta=a/b$, $\sigma_p=\omega_p/\nu_x$ with $\omega_p^2=q^2N/(\varepsilon_0\pi m\gamma^3\, ab)$

Example:

l=3, even

Isotropic:

$$D_{3,e}=8+\sigma_p^2\frac{12}{9-\sigma^2}-\sigma_p^4\frac{4\sigma^2\left(3-\sigma^2\right)}{\left(9-\sigma^2\right)^2\left(1-\sigma^2\right)^2}$$

Anisotropic:

$$\begin{aligned}
D_{3,e}\equiv&\left(1+\eta\right)^3+\\
&\frac{\sigma_p^2}{8}\Big[\frac{1-5\eta}{1-\sigma^2}+\frac{9+27\eta+24\eta^2}{9-\sigma^2}\\
&+\frac{(1-2\alpha)(1-2\eta^2/\alpha)(3+\eta)}{(1-2\alpha)^2-\sigma^2}+\frac{(1+2\alpha)(1+2\eta^2/\alpha)(3+\eta)}{(1+2\alpha)^2-\sigma^2}\Big]+\\
&\frac{\sigma_p^4}{8}\Big[\frac{-1}{(1-\sigma^2)^2}+\frac{3}{(1-\sigma^2)(9-\sigma^2)}\\
&+\frac{3(1-2\alpha)}{(9-\sigma^2)((1-2\alpha)^2-\sigma^2)}+\frac{3(1+2\alpha)}{(9-\sigma^2)((1+2\alpha)^2-\sigma^2)}\Big]\\
&=0
\end{aligned}$$

Im $\sigma>0$:
many resonant
instabilities

Recently resolved confusion about KV and non-KV modes & instabilities

„Gluckstern modes“ in symmetric beams known as KV artifact
- **Anisotropic**: we claimed some modes are „real“ (non-oscillatory) – not unambiguous
- Others emphasise highest frequency branches („fluid modes“ by Lund&Davidson“)

Key is „**coherent coupled mode coefficient**“ , generalizing Baartman‘s definition for symmetric beams to unsymmetric/anisotropic beams:

$$\omega = m\nu_z + l\nu_x + \Delta\omega = n$$

$$C_{mlk} = (m\nu_{0z} + l\nu_{0x} - \omega)/(m\Delta\nu_z + l\Delta\nu_x)$$

gives full coherent shift versus „incoherent coupled mode shift“

C_{202} (envelope)=0.75 -> 33% „intensity gain“

new finding: C>1 (shift past incoherent) are KV-specific „negative energy modes“ leading to „Gluckstern“ instabilities - need to be discarded

C<1 are positive energy modes of realistic beams -> anisotropy instabilities for C=1
-> most low-frequency branches also realistic (kinetic – not fluid)

Example:

$\nu_{0z}=4.115$ / $\nu_{0x}=4.35$

$\varepsilon_z / \varepsilon_x=2$

4^{th} order modes

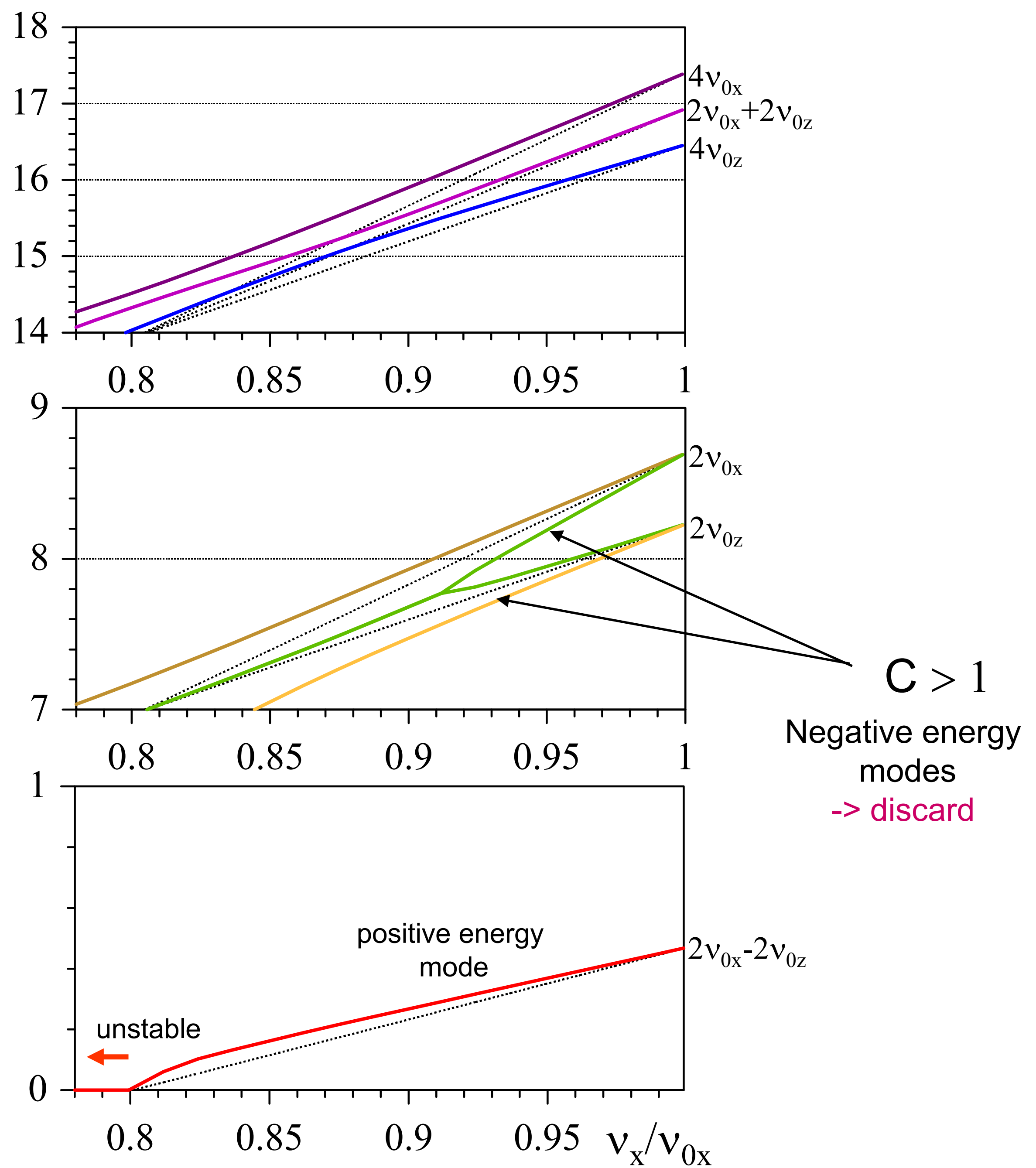
Examples for highest frequency branches:

2^{nd} order: $C_{20}= 0.65$

3^{rd} order: $C_{30} = 0.77$

4^{th} order: $C_{40} = 0.88$

trend: approaching 1

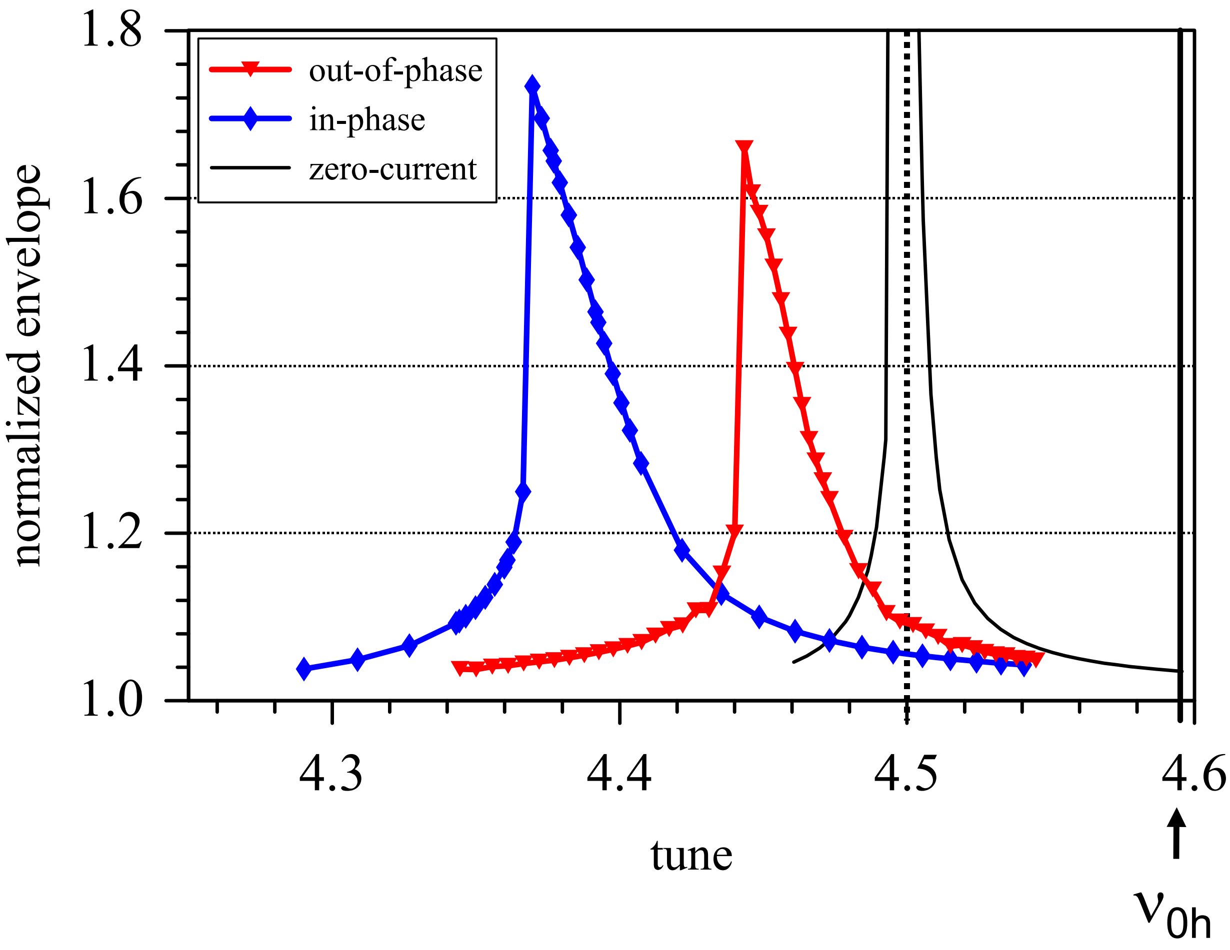


Applied to SNS ring: essentials from envelope equation

2^{nd} order even mode resonance: $2\nu_h + \Delta\omega = 9$

driven by gradient error near (fictitious) working point 4.5

Fedotov, Hofmann, PRSTAB 2002



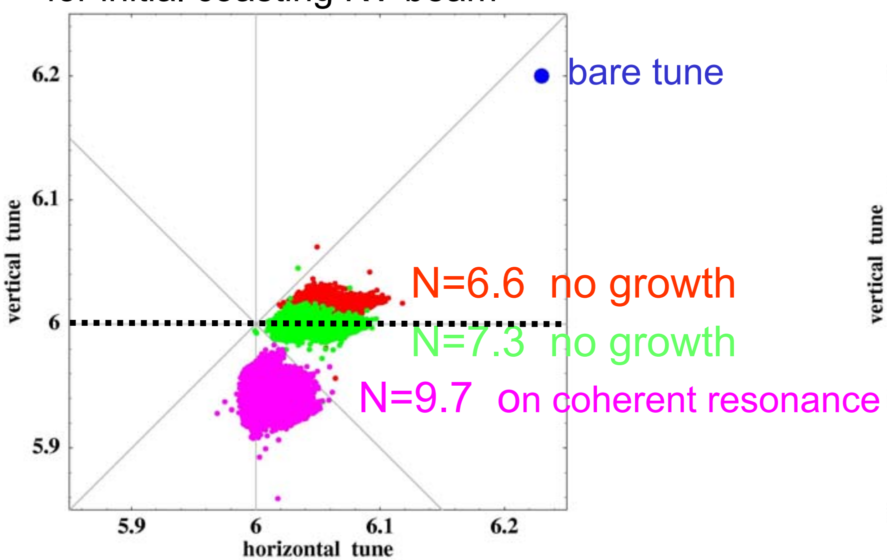
- **coherent shift of resonance**
- **different for breathing and quadrupolar mode**
- **envelope response limited due to de-tuning**

next: applied ORBIT code to SNS lattice

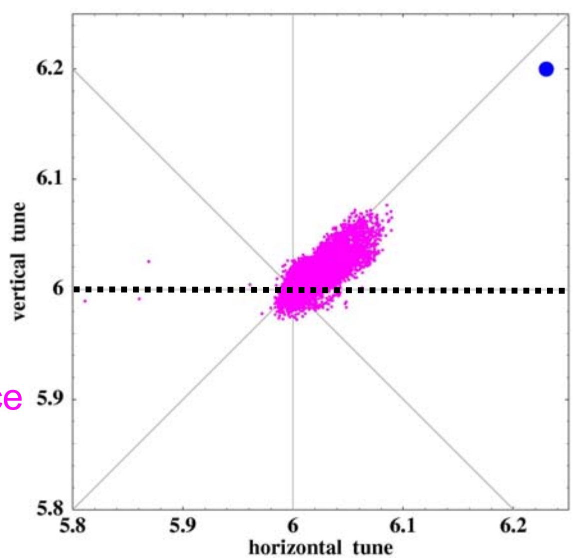
Found clear intensity gain due to the coherent shift of the 2nd order (systematic) resonance $2\nu_y+\Delta\omega=12$ (4 super periods)

intensity gain basically maintained for injection into bucket (< 1 synchrotron period)

single particle footprints (1st turn)
for initial coasting KV beam

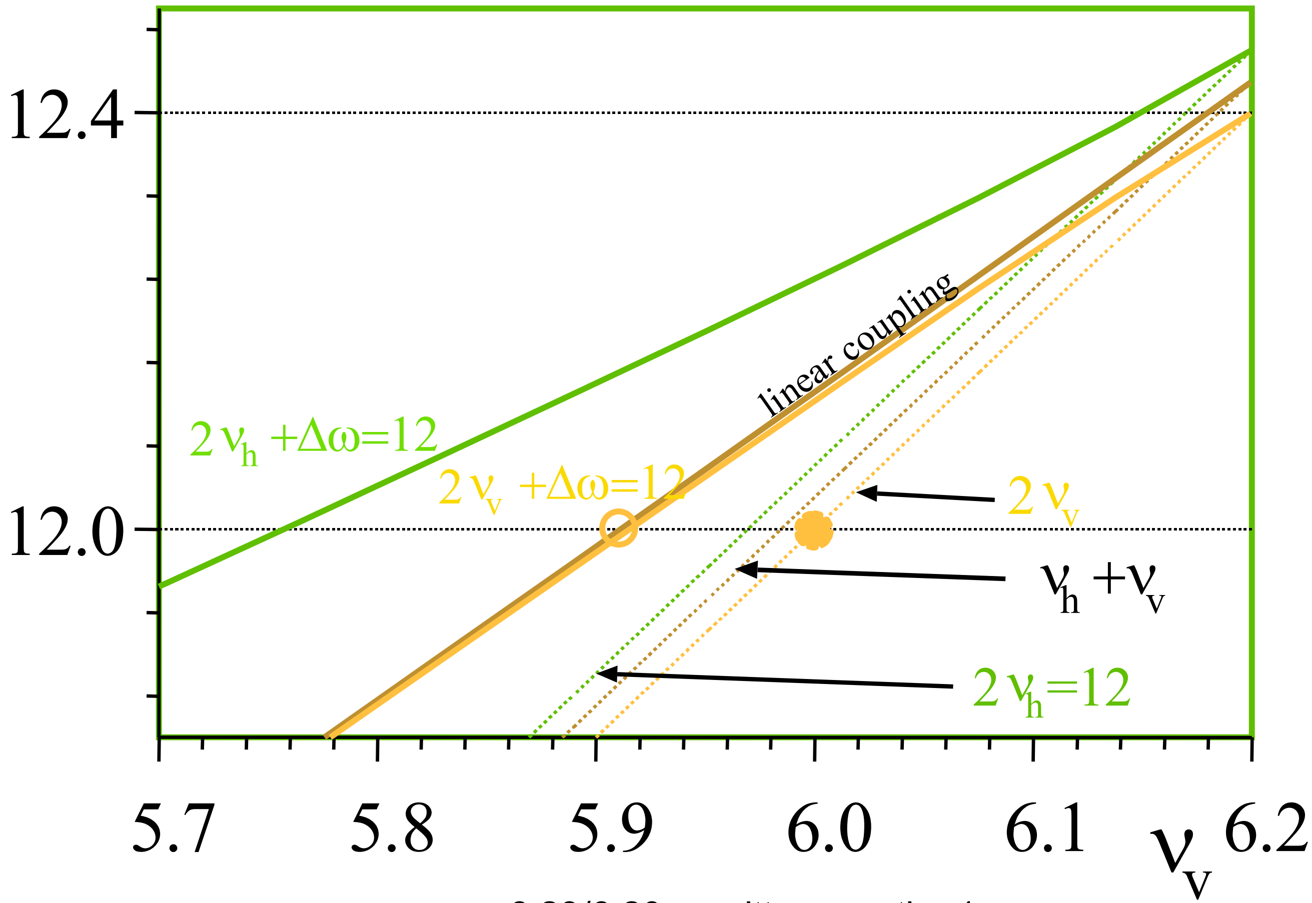


100 turns



Consistent with theoretical shift

$2\nu_v + \Delta\omega = 12$



ν_0=6.23/6.20; emittance ratio=1

Check 4^{th} order:

fourth order resonances much closer to $2\nu_v=12$:

$$4\nu_v+\Delta\omega=24$$

and:

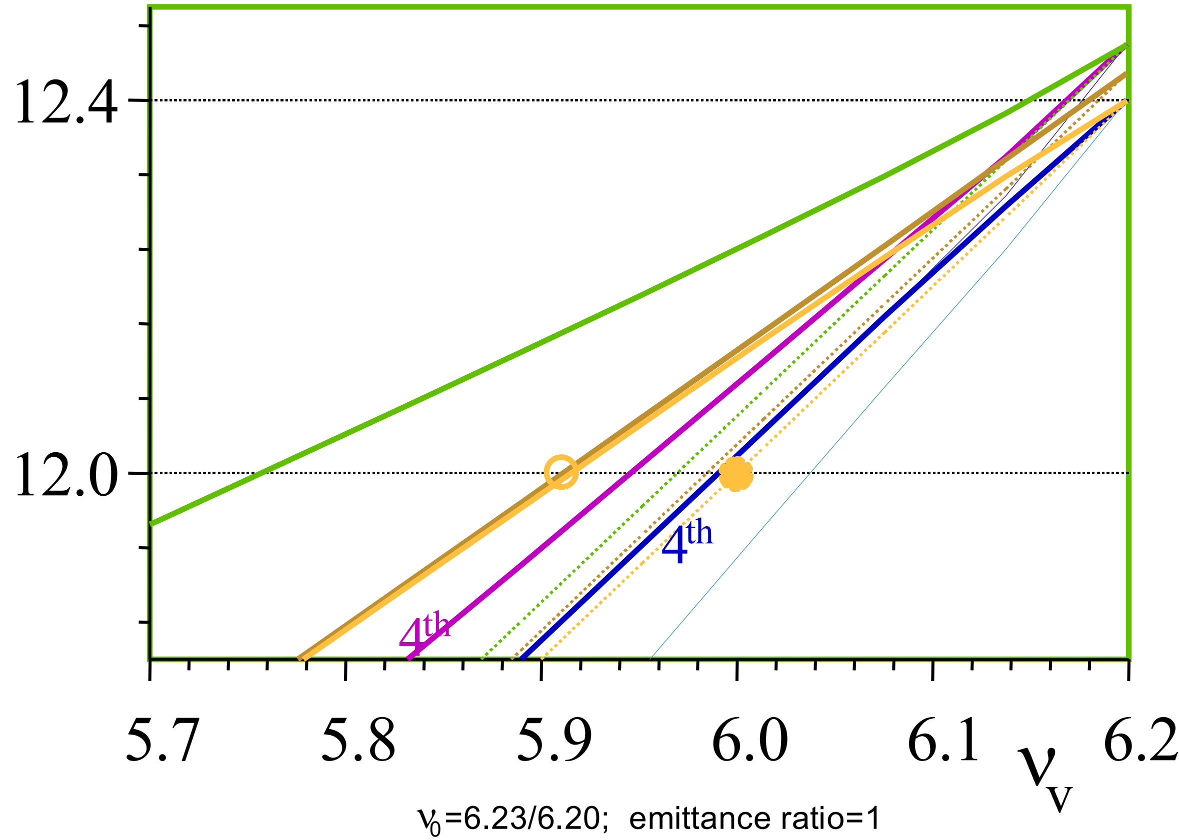
$$2\nu_v+\Delta\omega=12$$

does this take away „coherent advantage“?

first tests turning on fringe field octupoles in SNS: no

but: higher order might show up >> 1ms

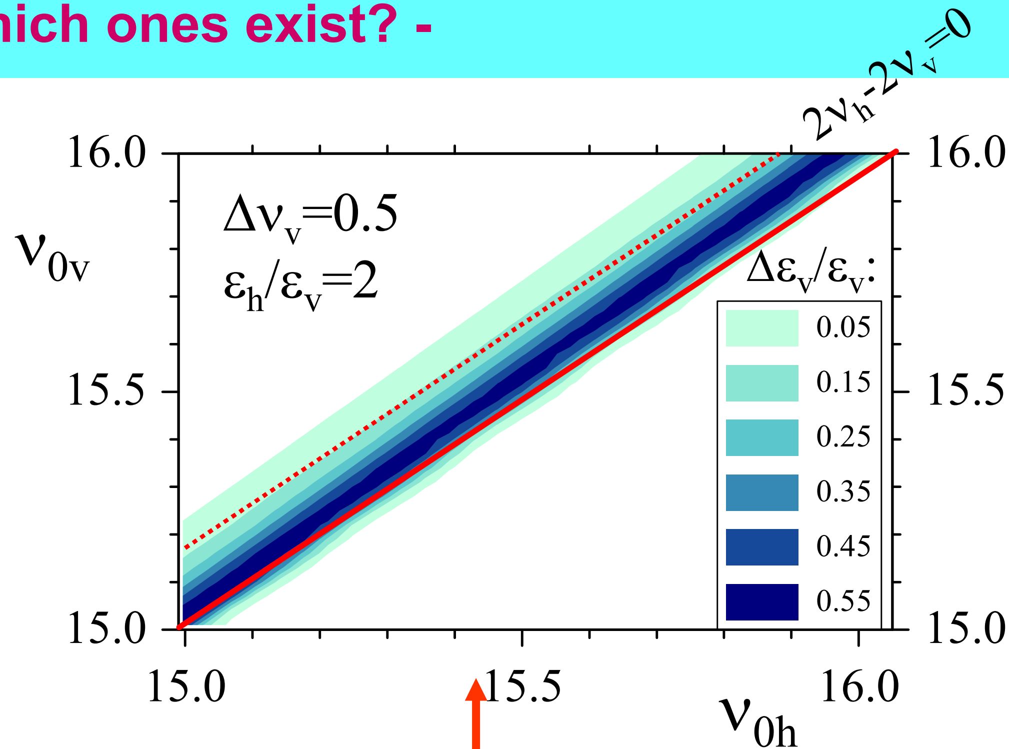
need careful study!



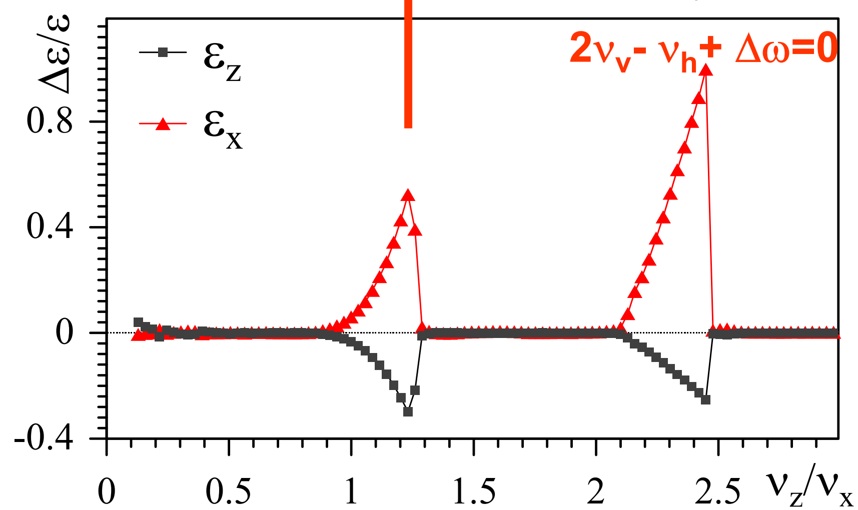
Higher order space charge driven resonances in linear lattice - which ones exist? -

„Montague“ resonance:
$2\nu_v - 2\nu_h + \Delta\omega = 0$

expect also:
$2\nu_v + 2\nu_h + \Delta\omega = nxN$
studied by Machida et al.



data taken from scan over stopband
using 2D PIC simulations for intitial waterbag:

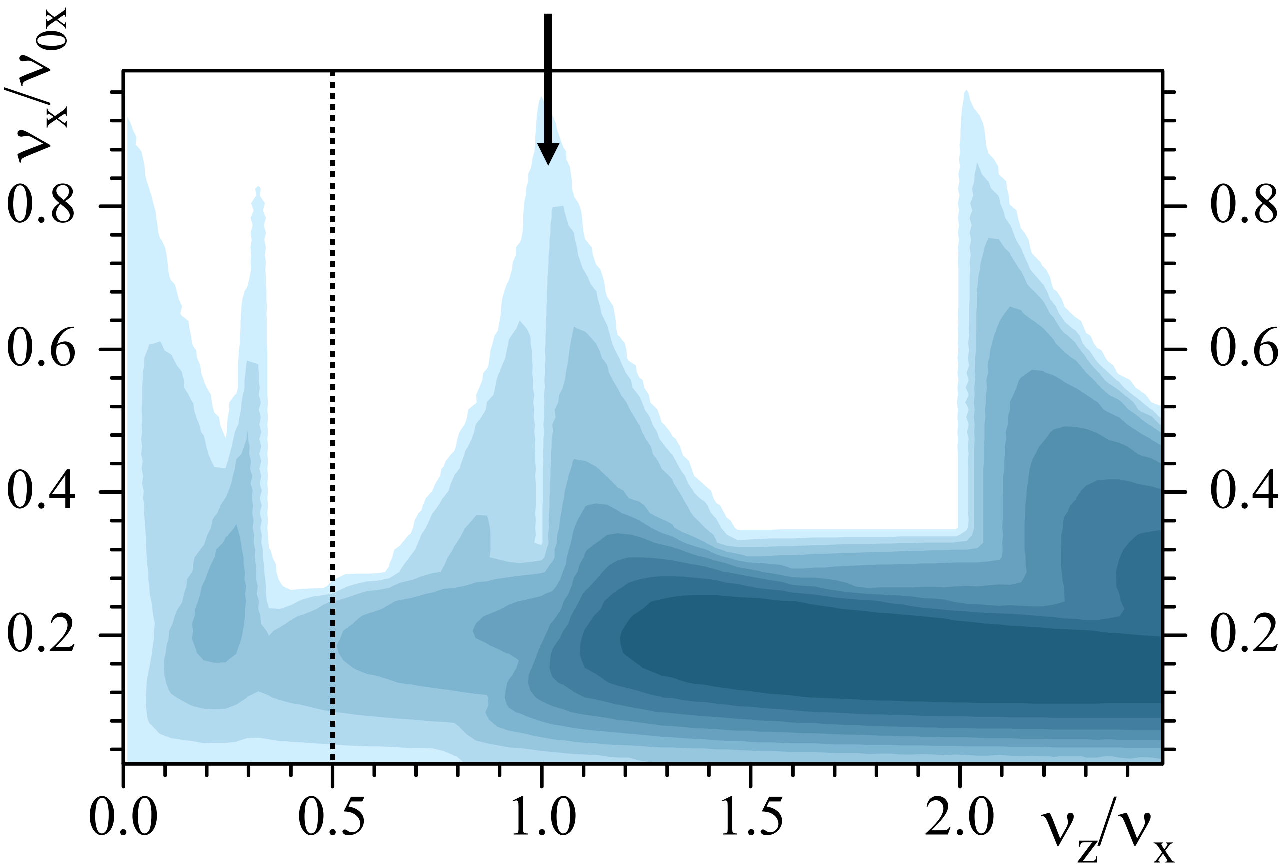


But: also find resonant growth (exponential) in third order due to space charge - we claim not only even sc-resonances matter!
Found $2\nu_h + \nu_v + \Delta\omega_3 = 12$ in pure linear lattice!

In Linacs same stopband responsible for emittance exchange

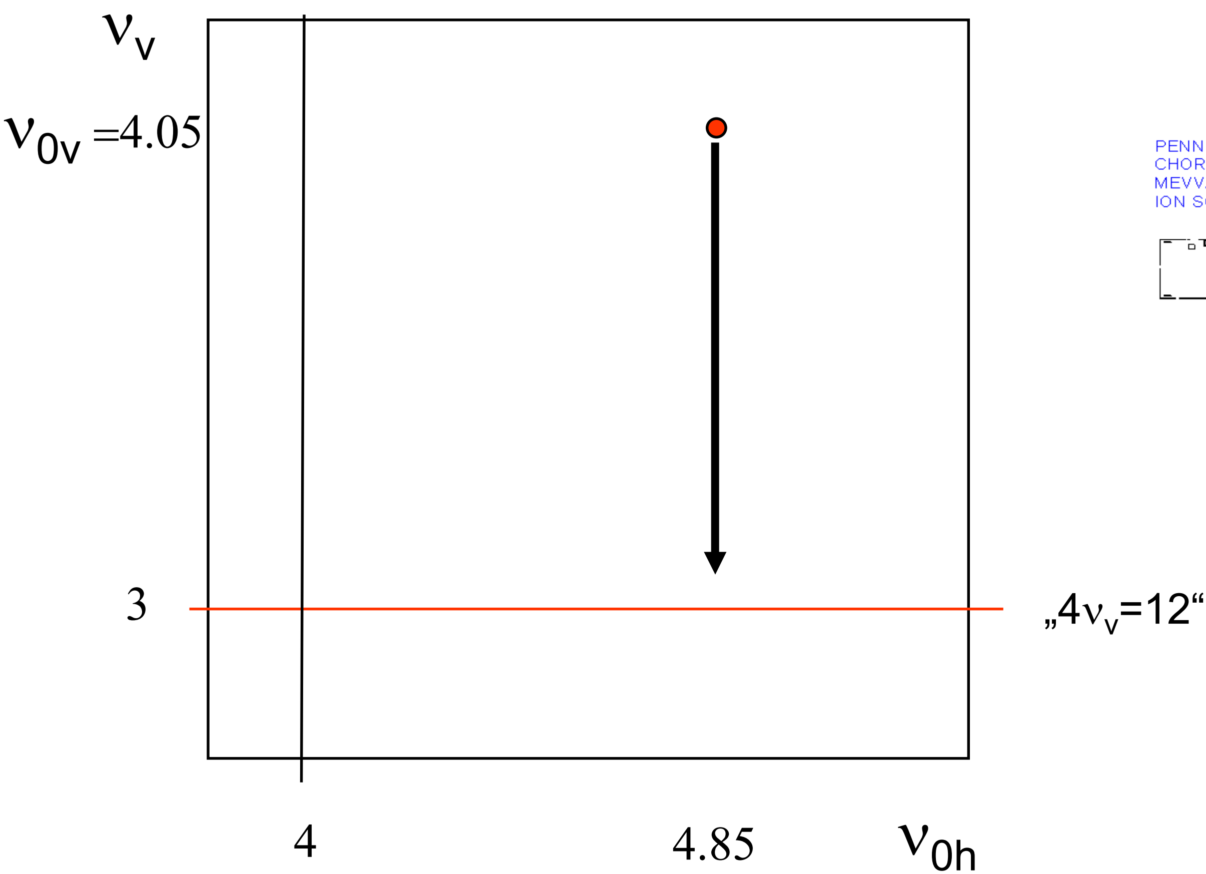
Montague, 1968
Hofmann, Boine-Frankenheim, PRL2001)
Sakai et al. PAC01

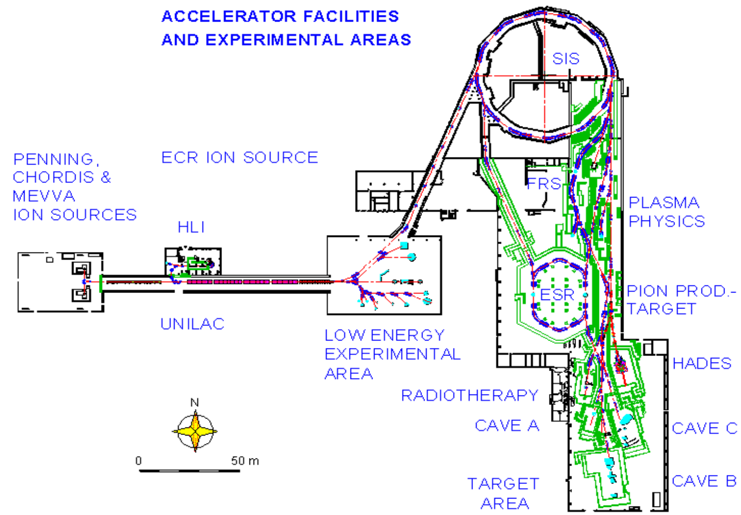
„Montague resonance“



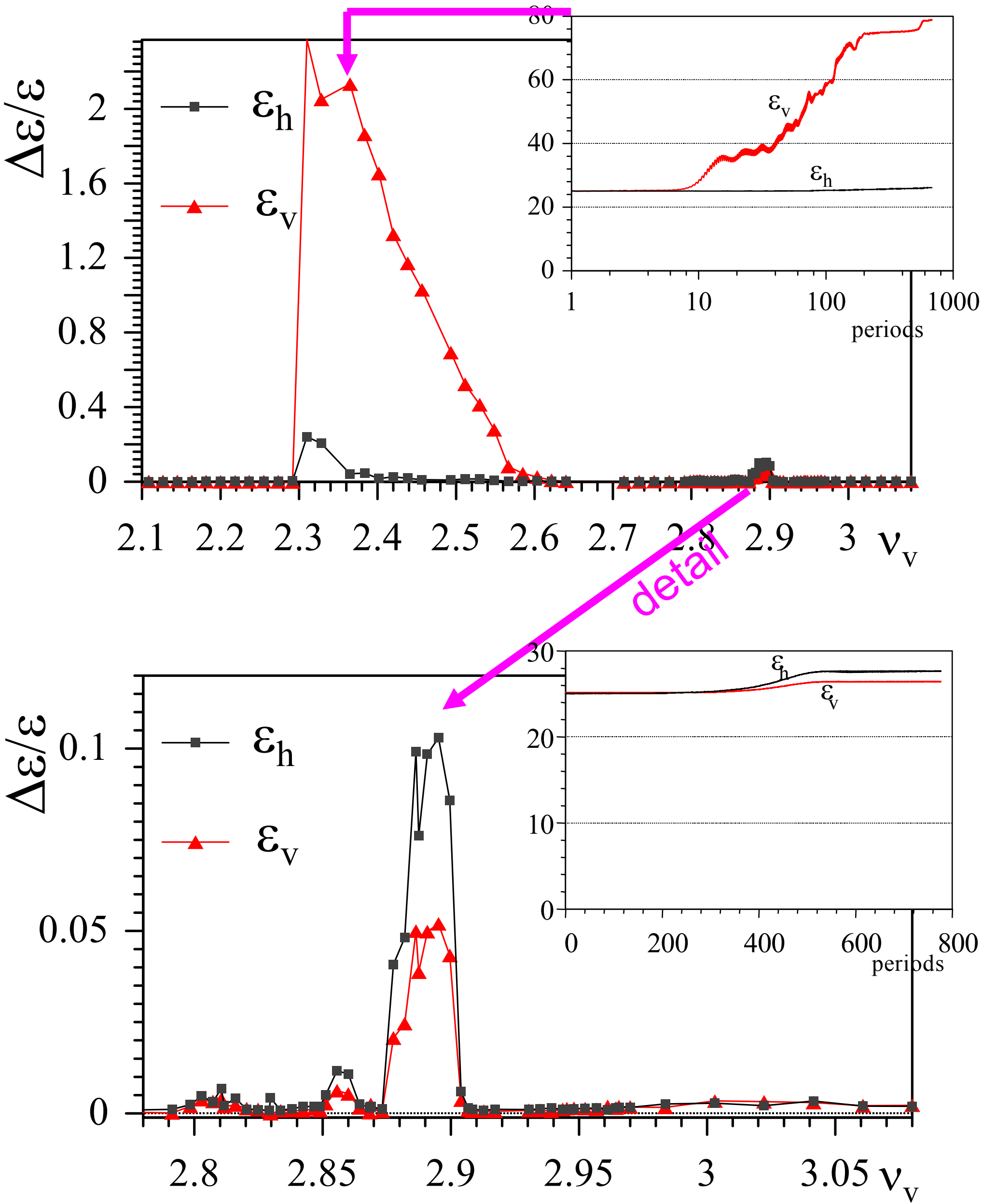
2d self-consistent PIC simulations for SIS18 lattice
(FODO with 12 super-periods)

using initial waterbag in 2D coasting beam/ 50.000 particles PIC





Found modes growing from noise = exponential instabilities



$2\nu_v + \Delta\omega_2 = 12/2$

envelope instability with $\sigma_0 > 90^0$

completely absent without space charge

Hofmann, Laslett, Smith, Haber, 1983

fourth order:

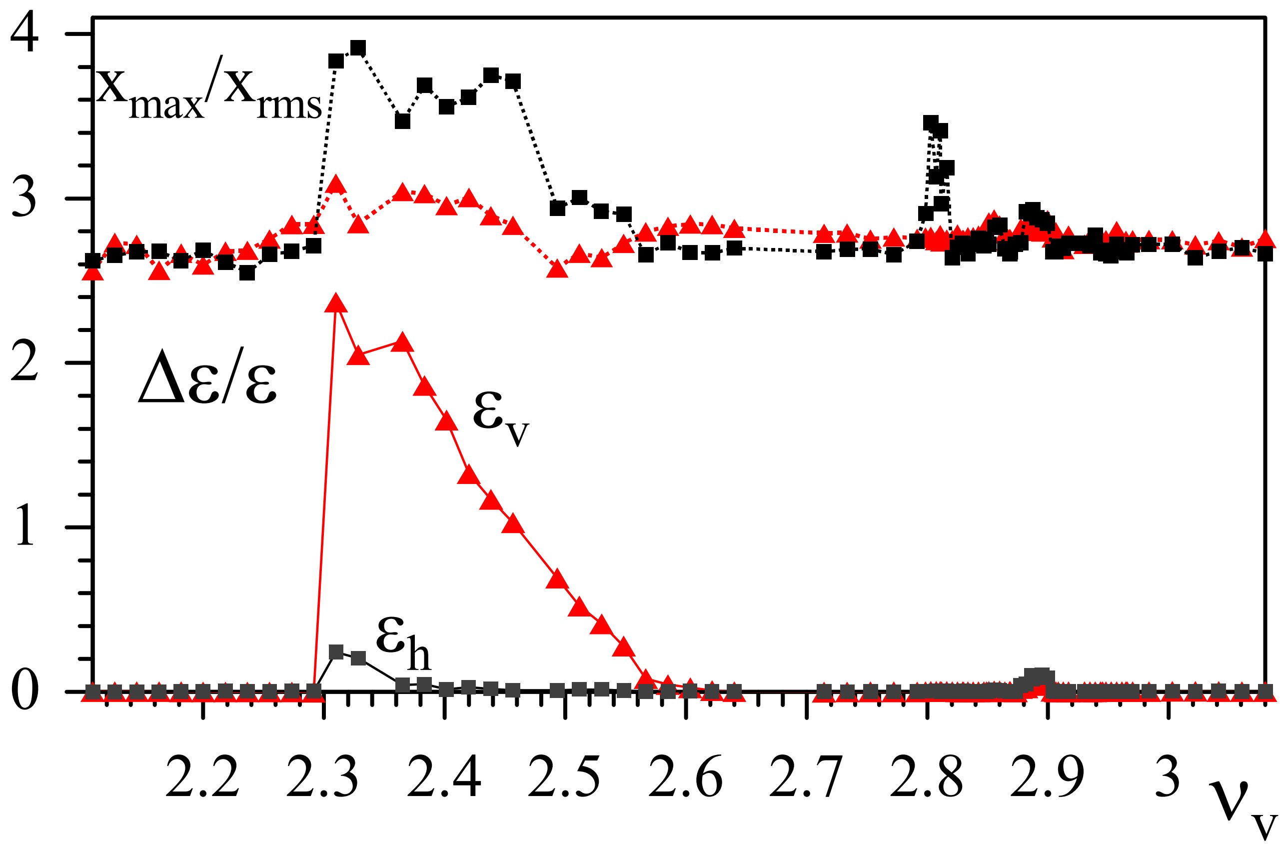
$4\nu_v + \Delta\omega_4 = 12$

$2\nu_v + \Delta\omega_4 = 12/2$

C~0.88 is in good agreement with theory

- **coherent shift of resonance**
- **emittance growth limited due to sc de-tuning**

Maximum radius



- **Rms emittance growth limited due to detuning**
- **Halo larger where less rms emittance growth**

Conclusions

1. Coherent resonance crossing & shift in second order
   - favourable coherent tune shift for ~ coasting beam (SNS)
2. For higher order modes found selection rule to discard „KV-modes“ (C_{mlk} >1)
   - coherent shift smaller in fourth order
3. Also found systematic sc driven instabilities $m\nu_h + l\nu_v + \Delta\omega = N/2$ in all orders
4. Pronounced self-limiting (de-tuning) effect of space charge in nonlinear (> 2nd order) resonance
   - suggests quite small resonance loss in 2D coasting beam or ideal barrier bucket -> significant resonance advantage of barrier bucket
5. Require „sufficiently fast“ synchrotron motion to explain observed losses in synchrotrons