Simulation of a space charge induced loss mechanism

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Overview

Motivation

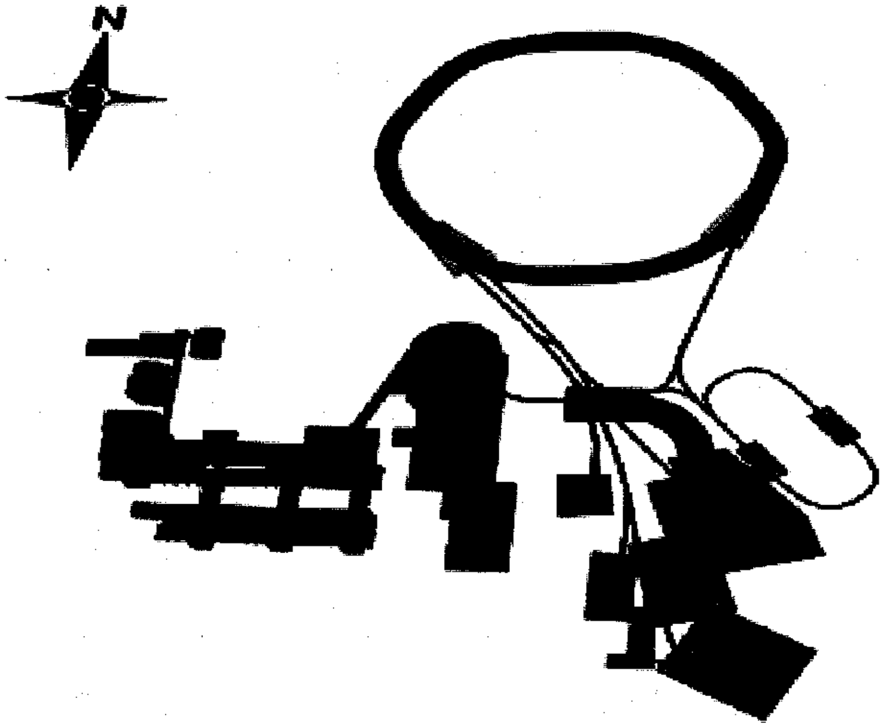
Long term tracking

Resonance crossing in 2D

Resonance crossing in 4D

Conclusion and Outlook

Motivation: GSI future project

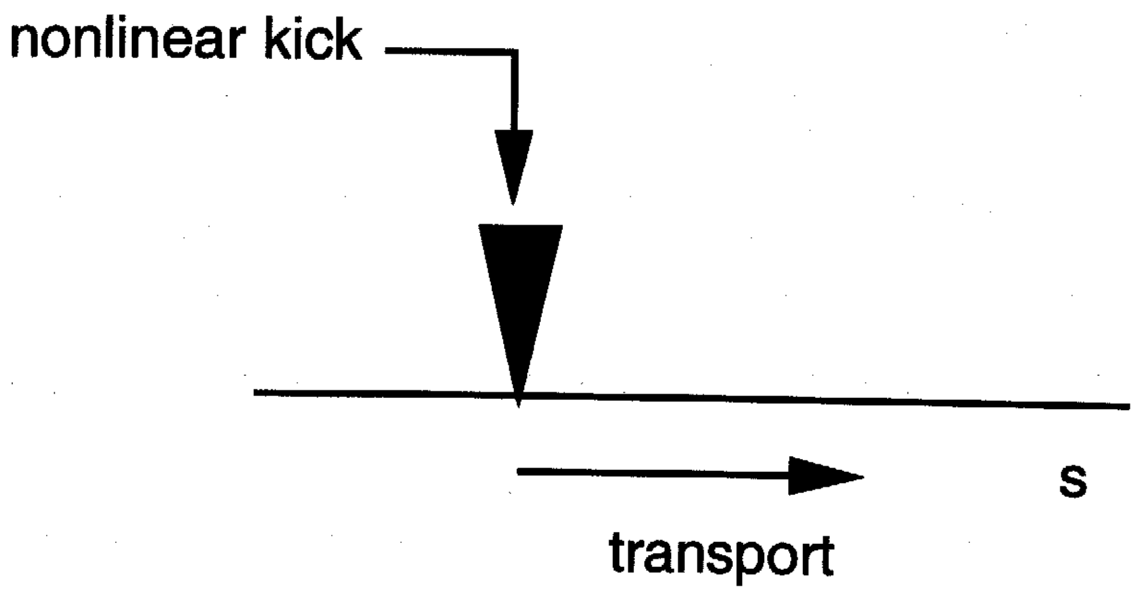


Accumulation of four SIS18 batches of U +28 at 96 MeV/u in the SIS100 in about 1 second.

This takes hundreds thousand turns

Maximum tuneshift of a bunched beam is 0.2

Long term tracking $\Longleftrightarrow$ symplectic algorithms



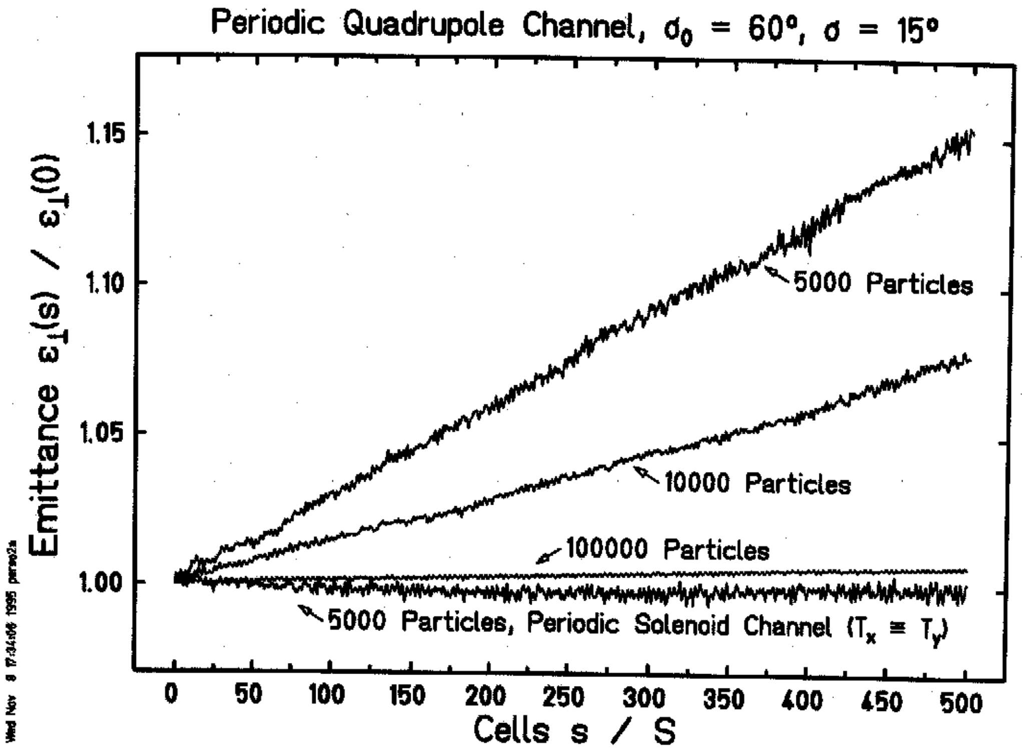
example:

$$\begin{pmatrix} \sqrt{\frac{\beta}{\beta_0}}(\cos\Delta\psi + \alpha_0\,\sin\Delta\psi) & \sqrt{\vphantom{\beta}}\sin\Delta\psi \\ -\frac{1}{\sqrt{\beta\,\beta_0}}[(\alpha-\alpha_0)\cos\Delta\psi + (1+\alpha\,\alpha_0)] & \sqrt{\frac{\beta}{\beta_0}}(\cos\Delta\psi - \alpha\,\sin\Delta\psi) \end{pmatrix} \begin{pmatrix} x \\ x' + k_2 x^2 \end{pmatrix}$$

Space charge forces

Poisson solver, needs big number of particles in order to obtain a noise free space charge force

The algorithm produces unavoidably numerical noise which causes emittance growth (J. Struckmeier Phys. Rev. E 54, 830–837)



Equation of motion: 2D example

equation of motion: $$x'' + \left(\frac{q_0}{R}\right)^2 x = 2K\frac{x}{r^2}(1 - e^{-\frac{x^2}{2\sigma^2}})$$

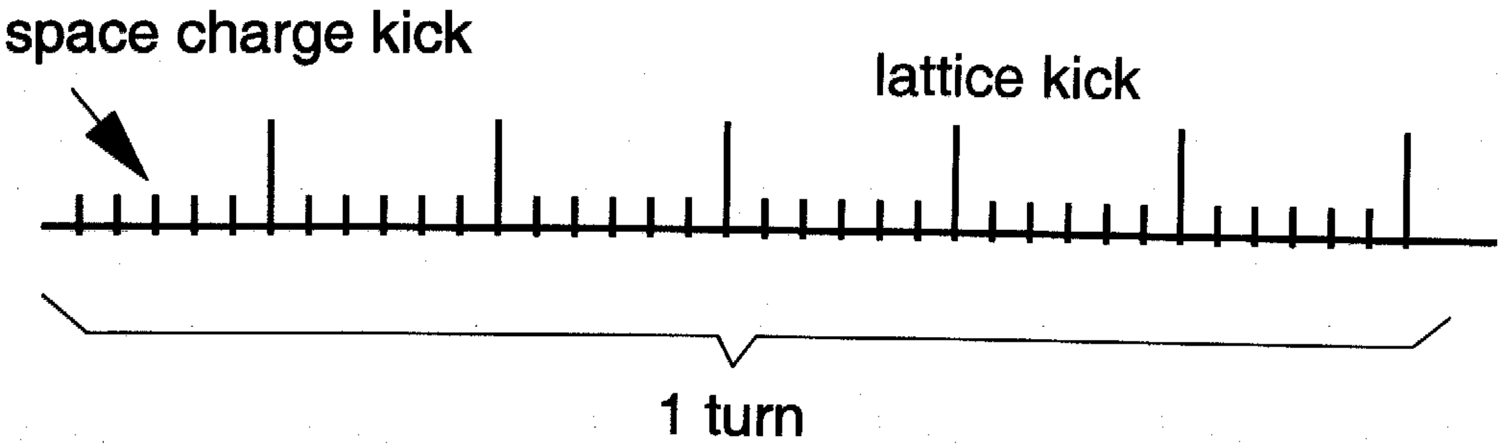
$$C = 2\pi R \qquad x = \tilde{x}\sigma \qquad s = C\tilde{s}$$

$$\tilde{x} = \frac{x}{\sigma} \qquad \dot{\tilde{x}} = x'\frac{C}{\sigma}$$

dimensionless equation

$$\boxed{\ddot{\tilde{x}} + (2\pi q_0)^2\tilde{x} = (2\pi)^2(2q_0\Delta q - \Delta q^2)\frac{\tilde{x}}{\tilde{r}^2}(1 - e^{-\frac{\tilde{x}^2}{2}})}$$

perveance: $$K = \left(\frac{\sigma}{C}\right)^2(2\pi)^2(2q_0\Delta q - \Delta q^2)$$



$$\begin{pmatrix} x \\ x' \end{pmatrix} = \begin{pmatrix} \cos(2\pi \frac{q_0}{N}) & \frac{1}{2\pi q_0} \sin(2\pi \frac{q_0}{N}) \\ -2\pi\, q_0 \sin(2\pi \frac{q_0}{N}) & \cos(2\pi \frac{q_0}{N}) \end{pmatrix} \begin{pmatrix} x \\ x' + f_n(x) \end{pmatrix}$$

Nonlinearites:

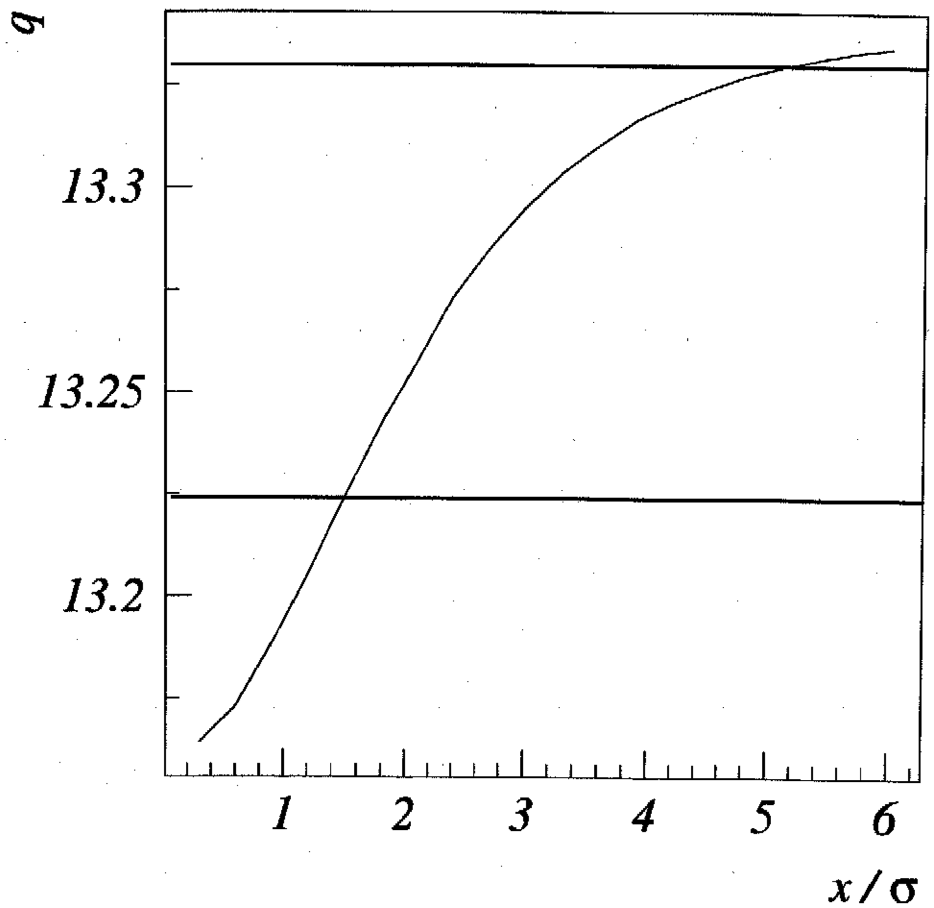
space charge: $$f_n(x) = \frac{1}{N}(2\pi)^2(2q_0\Delta q - \Delta q^2)\, x\, \frac{1 - e^{-x^2/2}}{x^2}$$

Lattice: $$f_n(x) = a_2 x^2 \qquad f_n(x) = a_3 x^3$$

Space charge induced tune spread

$q_0 = 13.36$

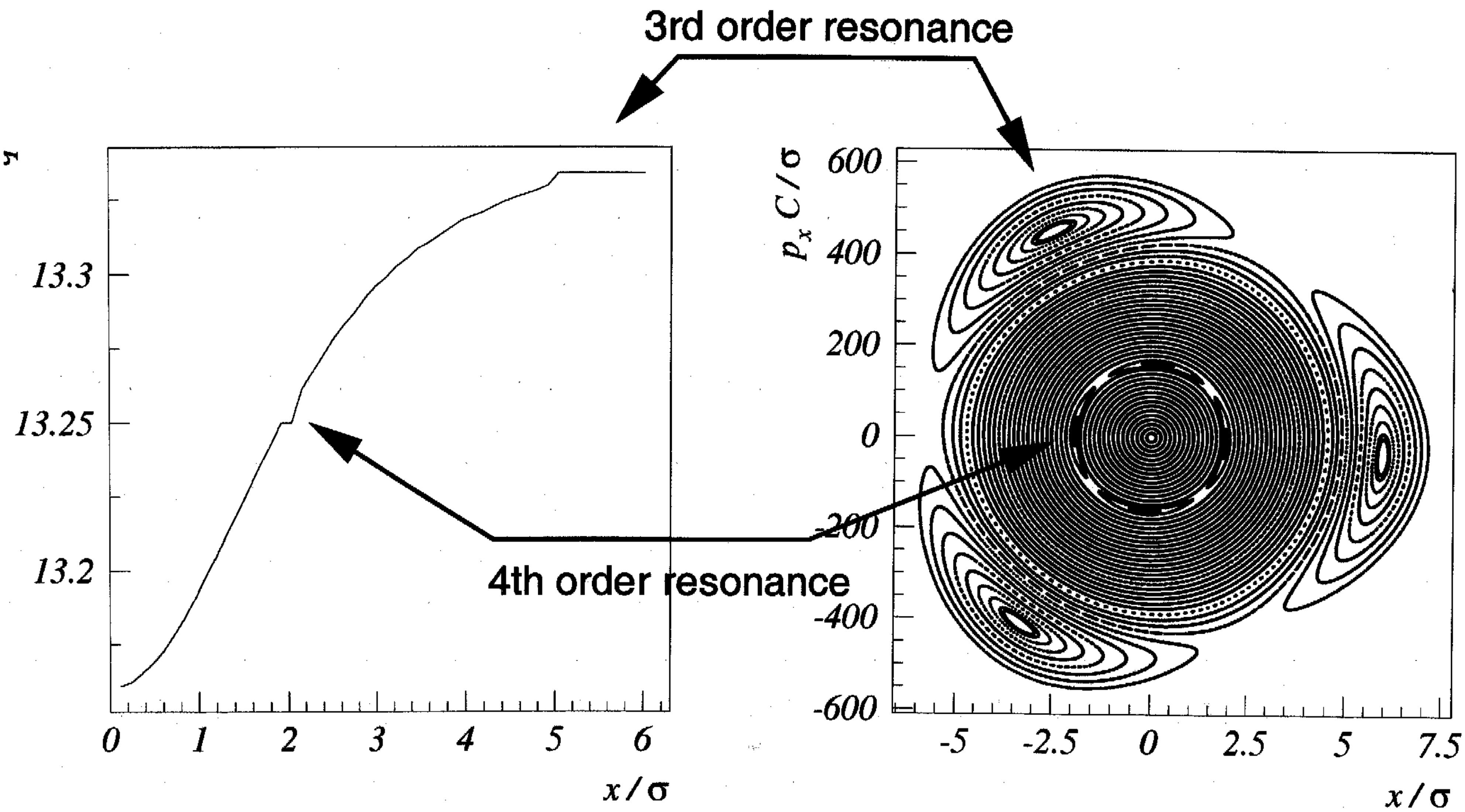
$\Delta q = 0.2$



3 q = 40

3 q = 53

Lattice error resonances



20 sext. error $a_2 = 0.01$ 53 oct. errors $a_3 = 0.005$

the position of the fixed points are determined by the resonance condition

the area of the islands increases when resonance is reached far from the beam center

The change of the position the resonance can be obtained in two ways:

1 letting fixed the beam current and changing the bear tune

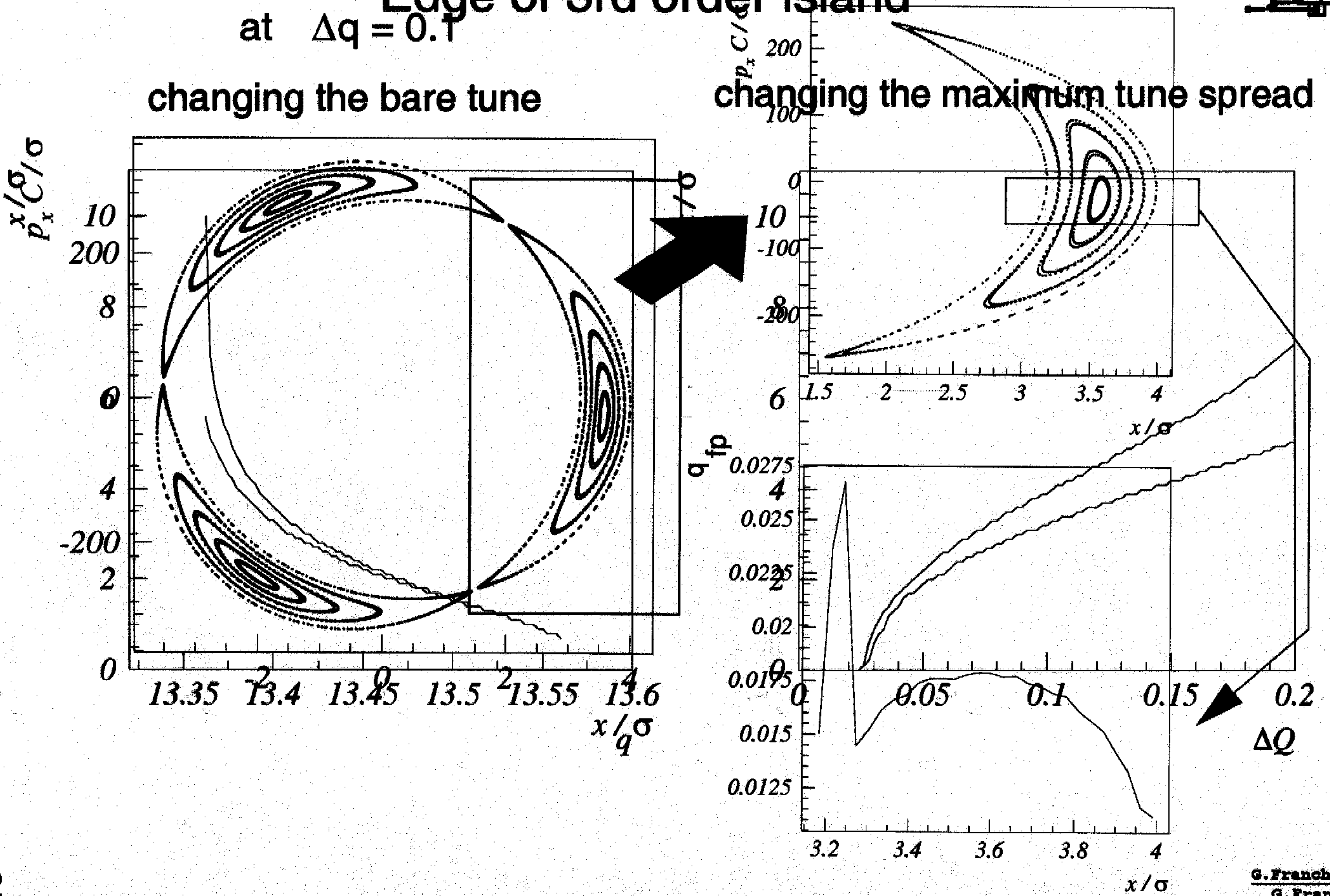
2 letting fixed the bear tune and changing the beam current

Edge of 3rd order island

at $\Delta q = 0.1$

changing the bare tune

changing the maximum tune spread



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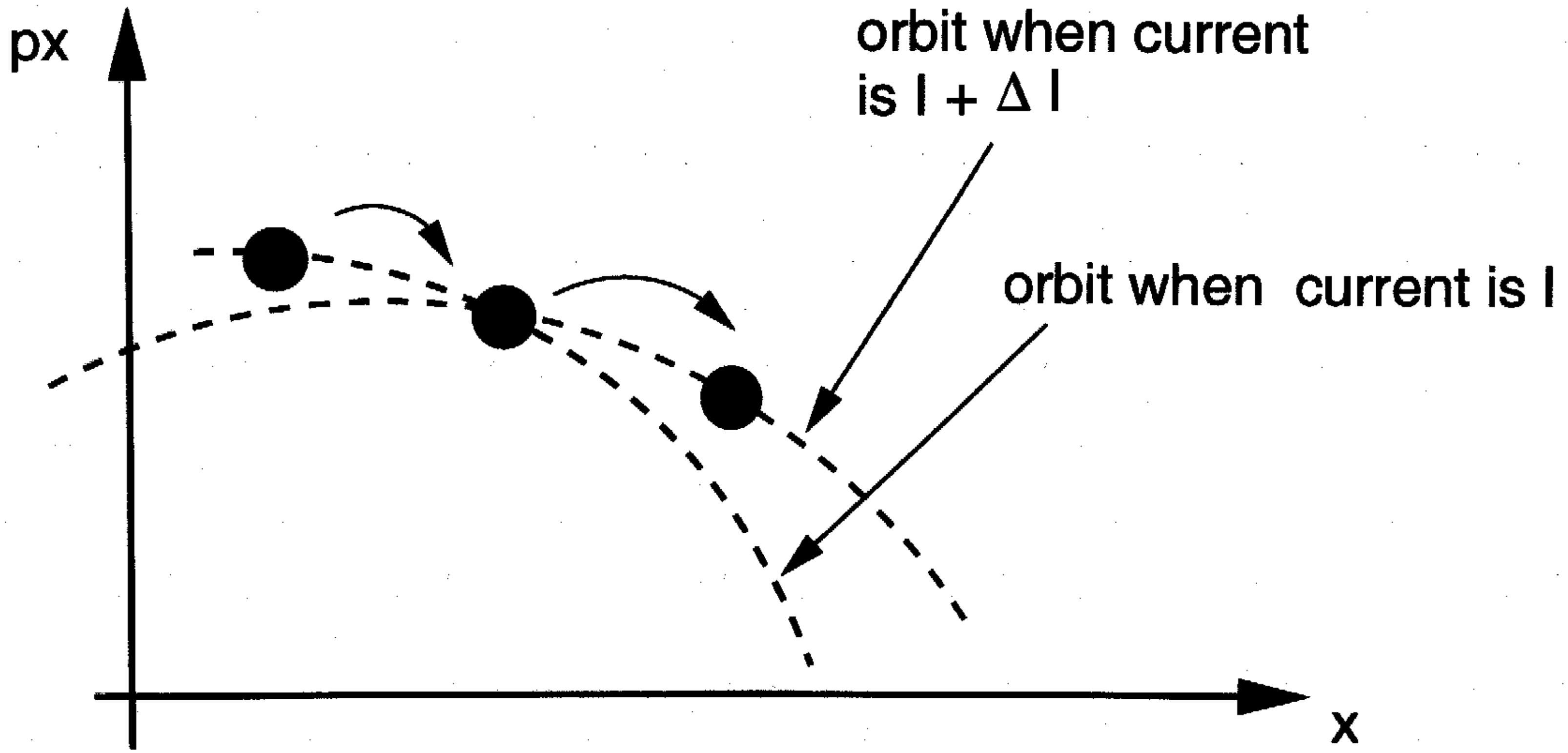
Dynamic change of space charge

During each turn the particle follows its own orbit



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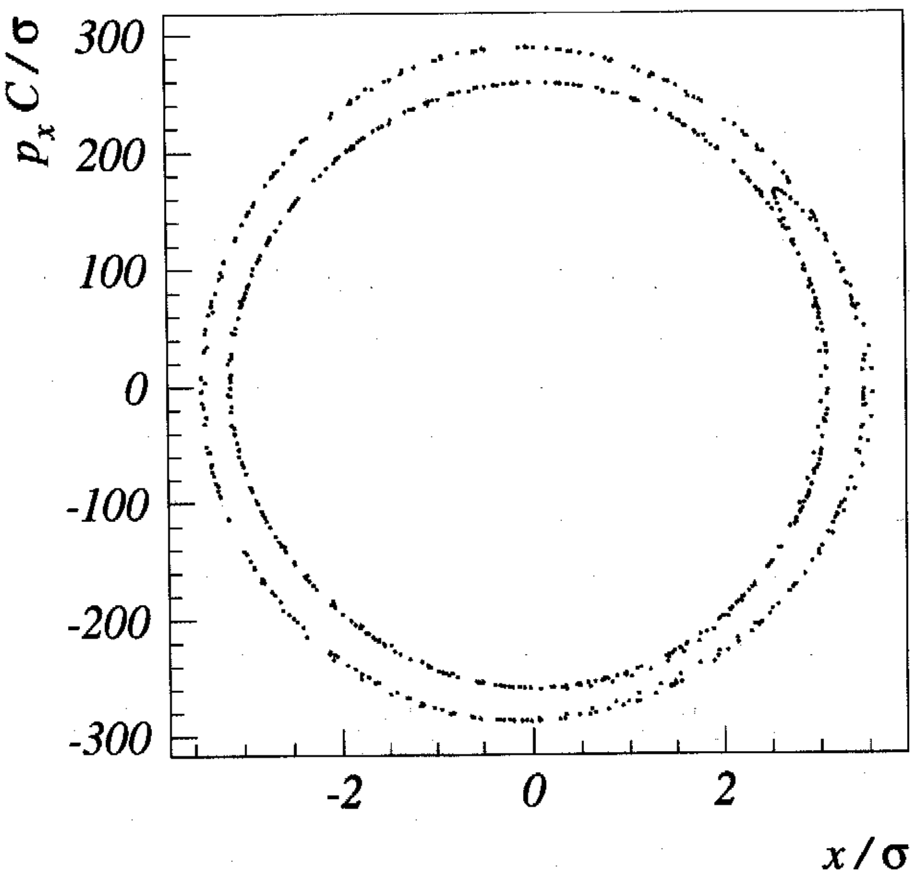
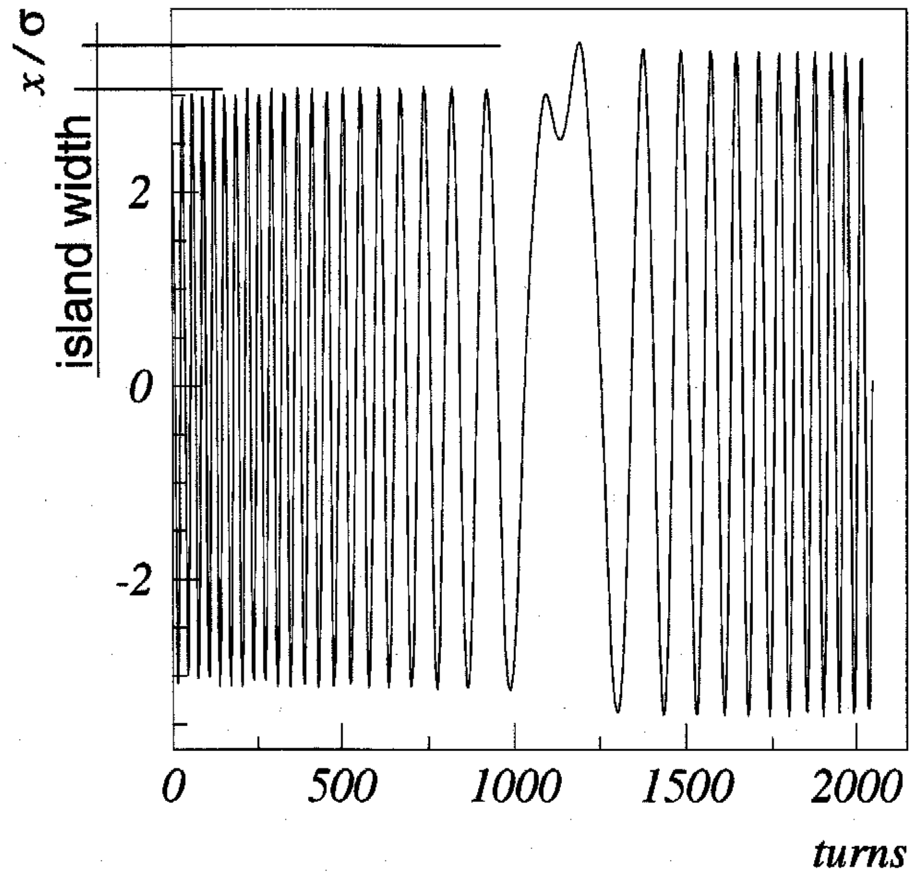
Island coming from outside the particle orbit

x = 3
px = 0

q = 13.36

current ramp −0.2 / 2048 [Δ q/turns]





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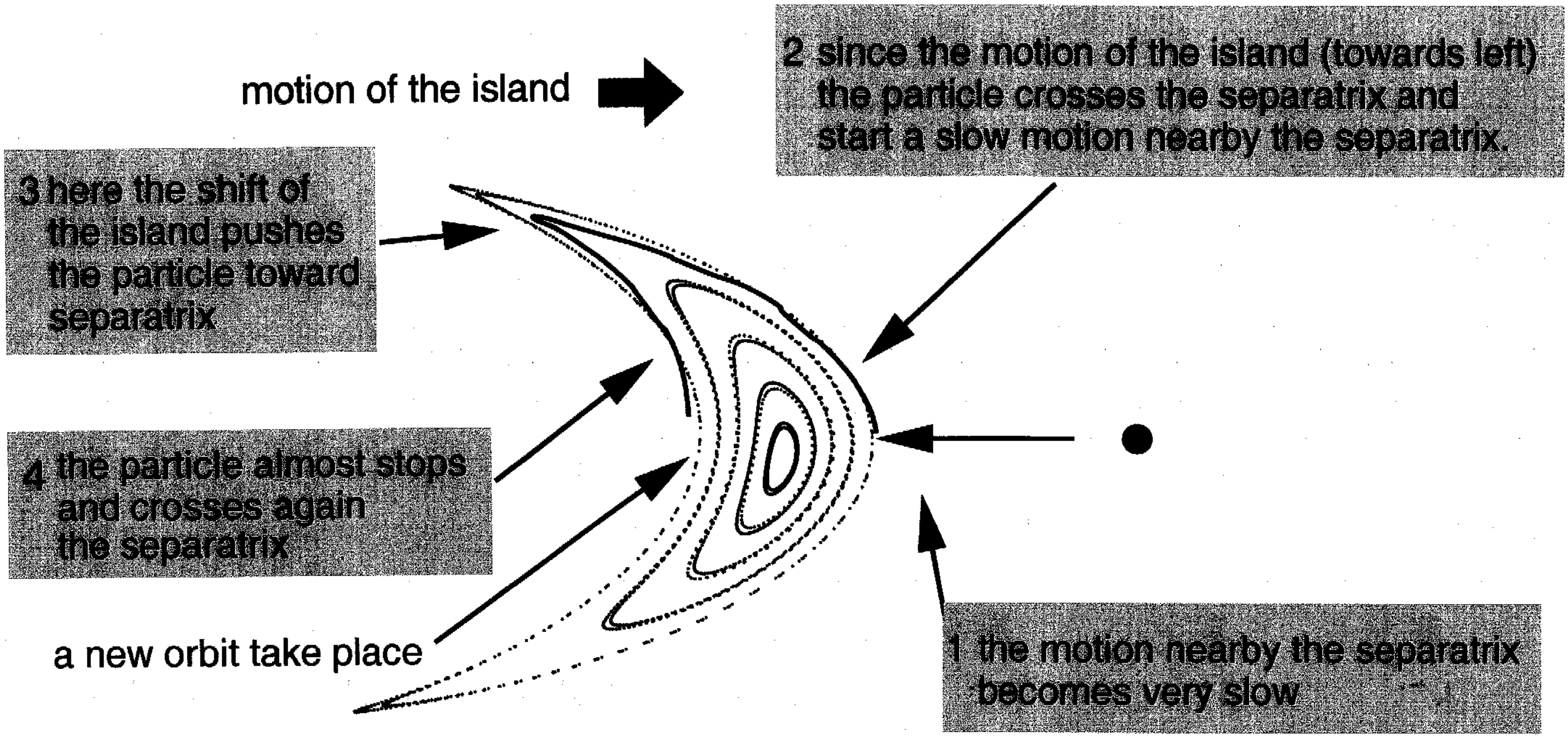
Slow increasing current



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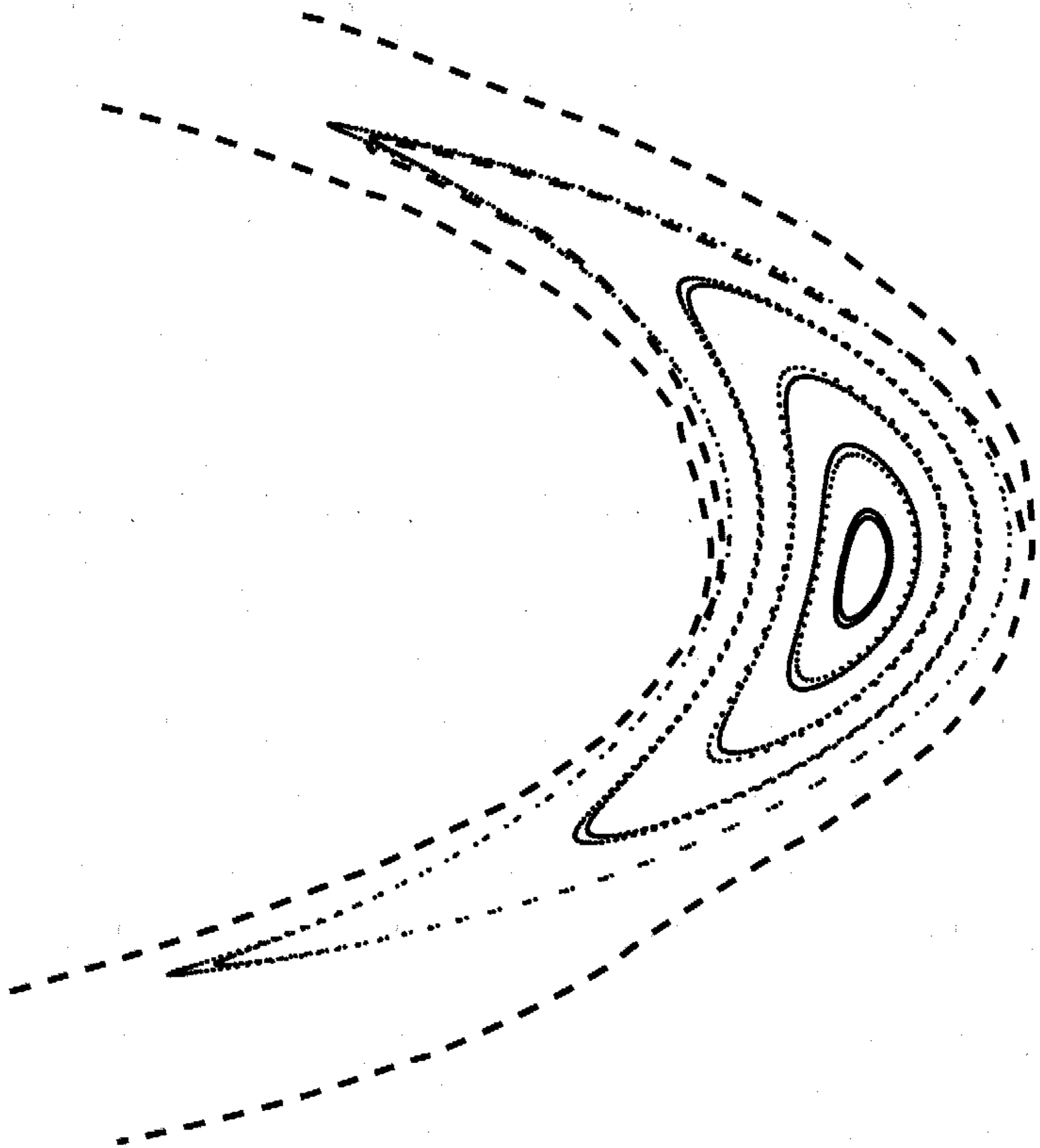
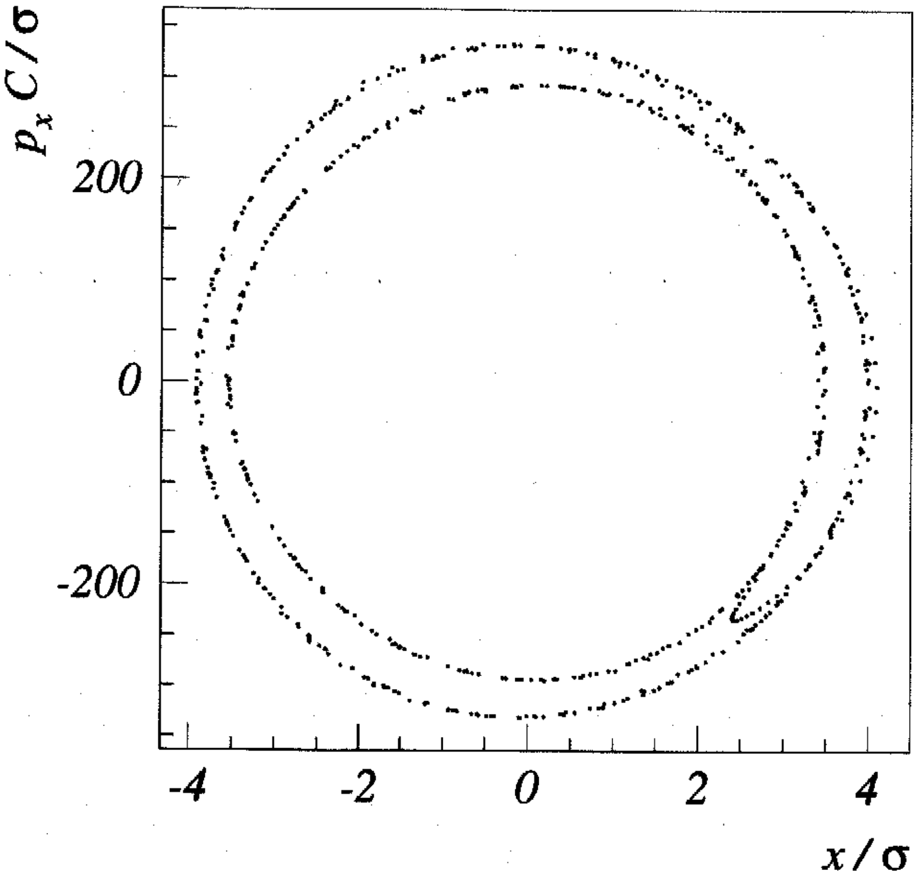


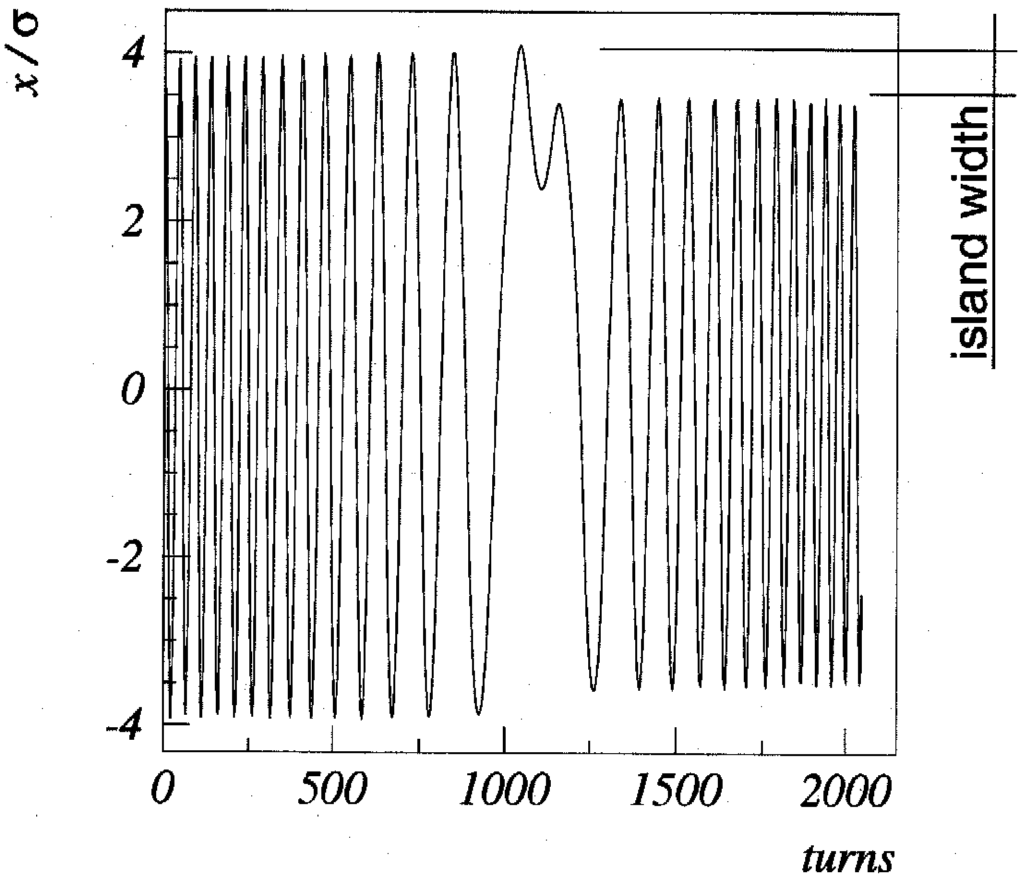
island coming from inside particle orbit

x = 4
px = 0

q = 13.36

current ramp 0.2 / 2048 [Δq/turns]



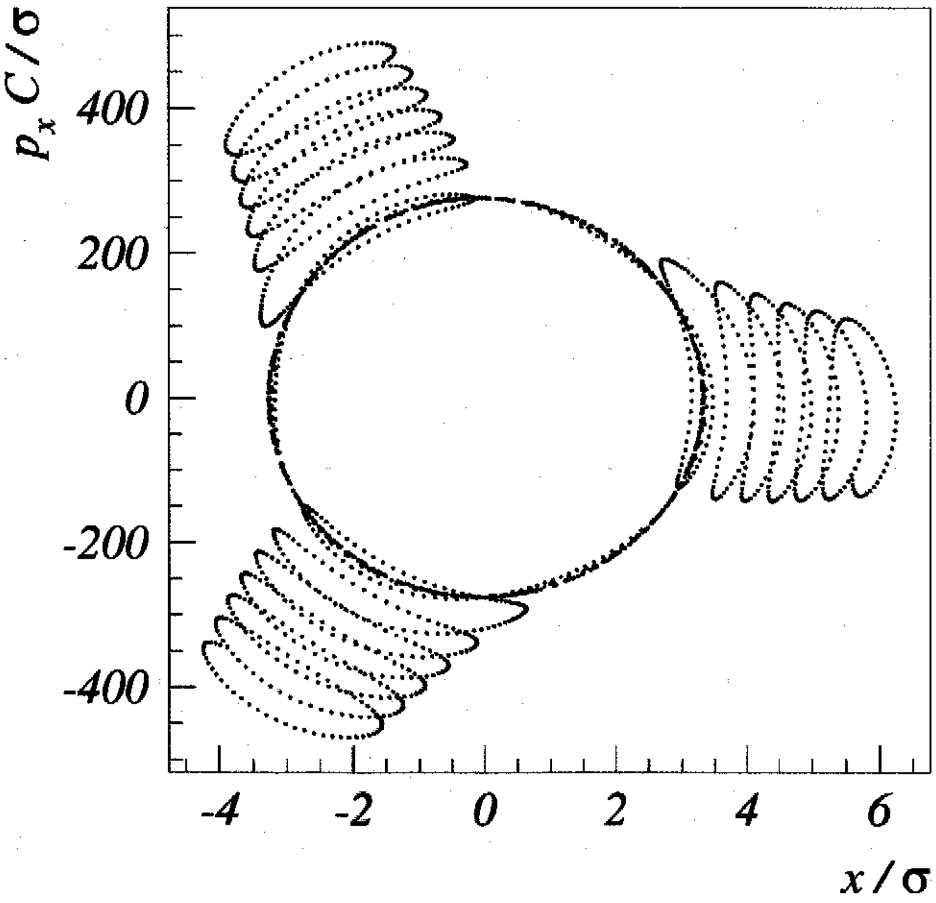


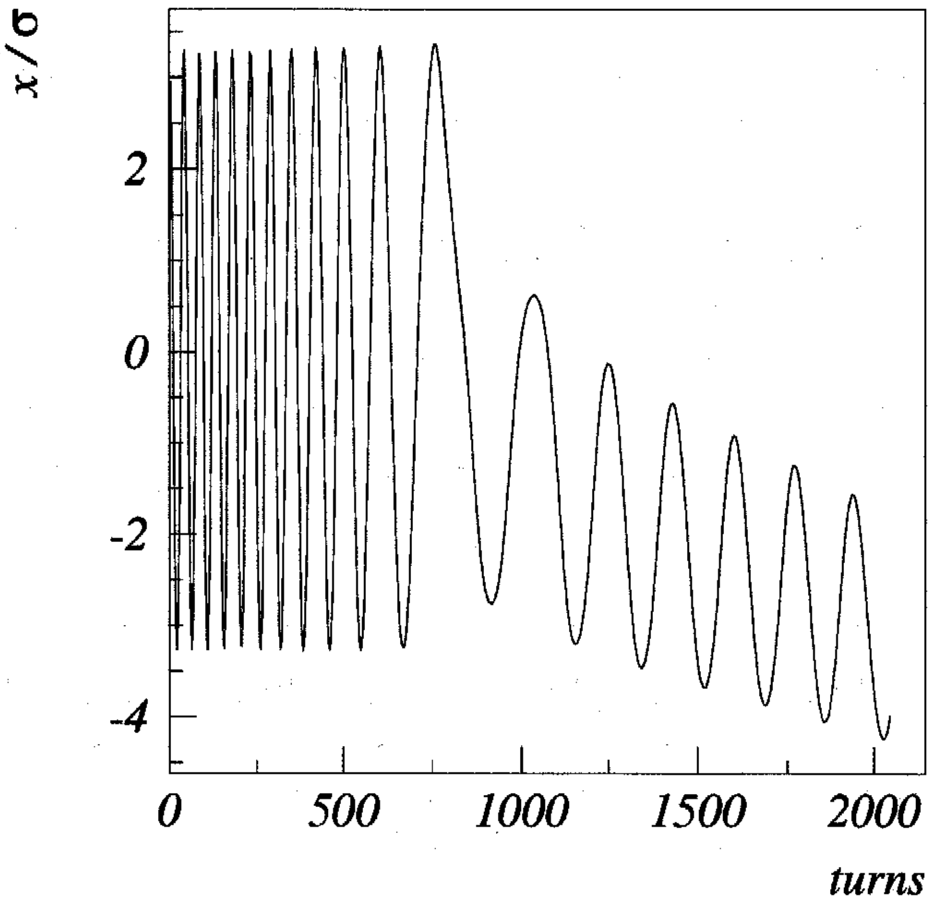
Trapping of particle into the island

x = 3.3
px = 0

q = 13.36

current ramp 0.2 / 2048 [Δq/turns]





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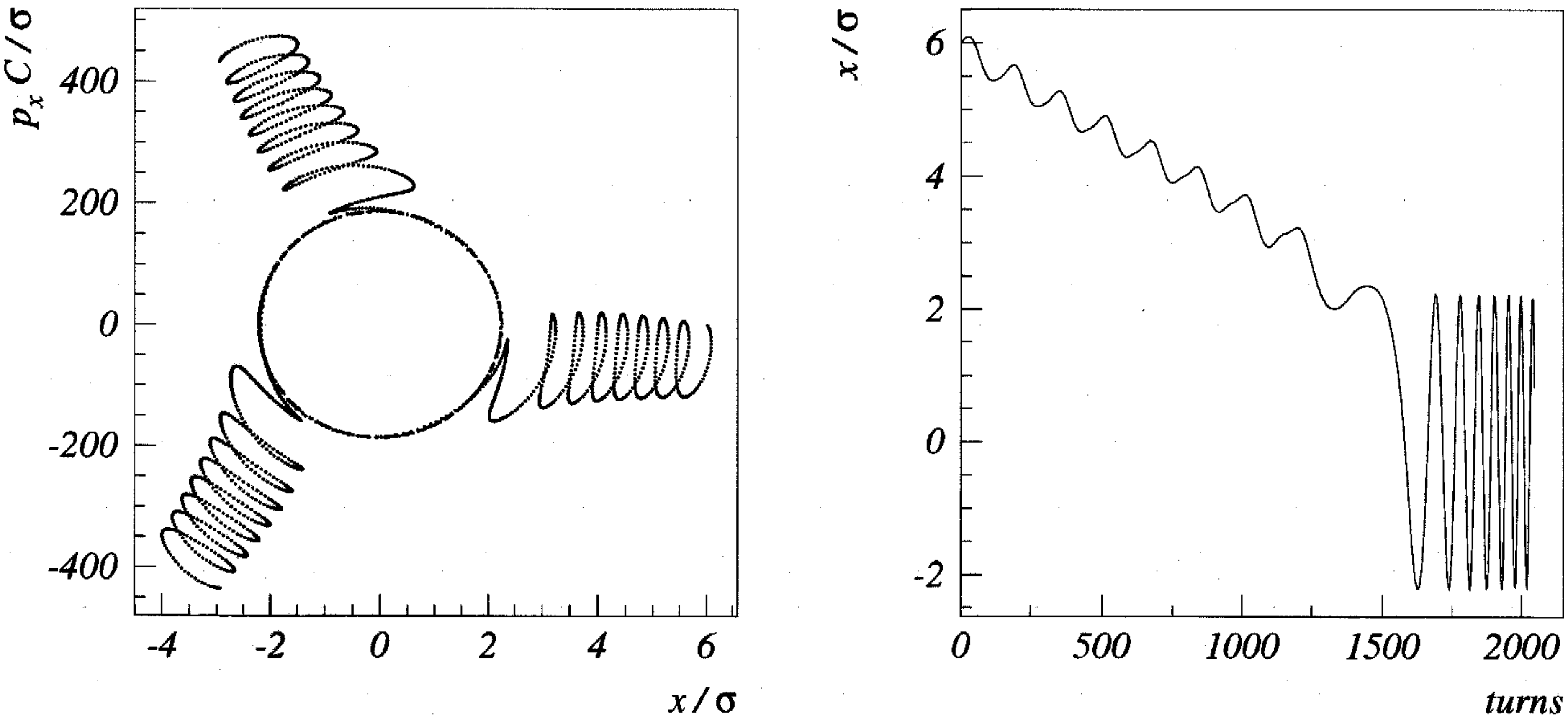
For a negative rate the capture process inverts the effect

initial coordintes x=6, px = 0   from Dq = 0.02 to Dq = 0



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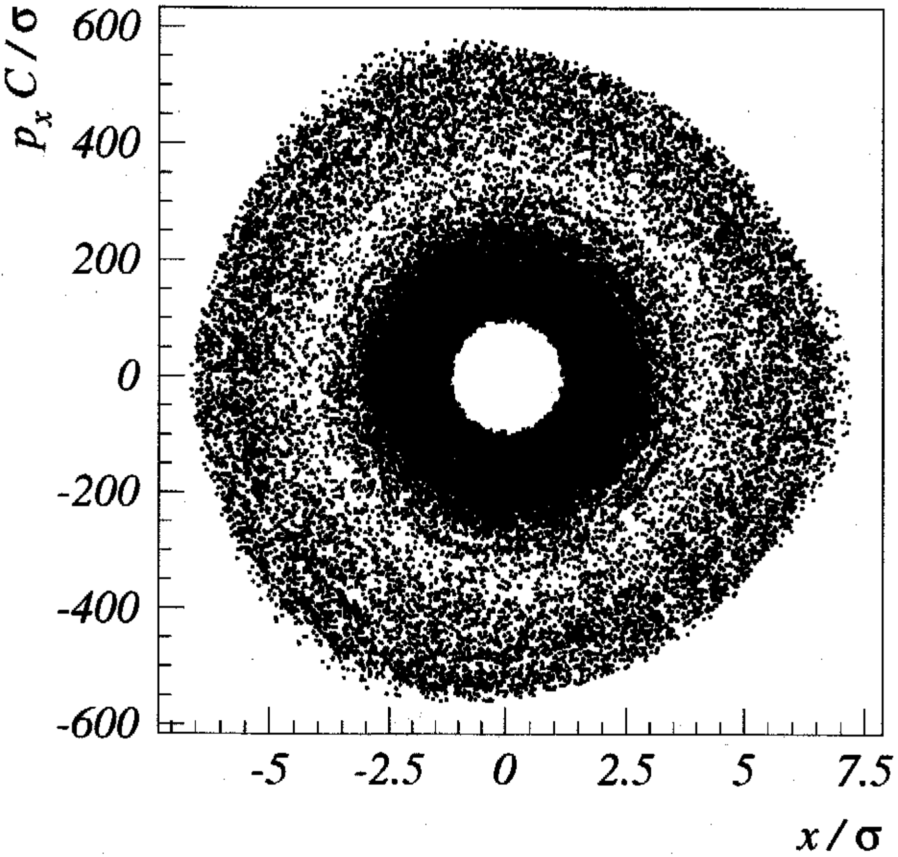
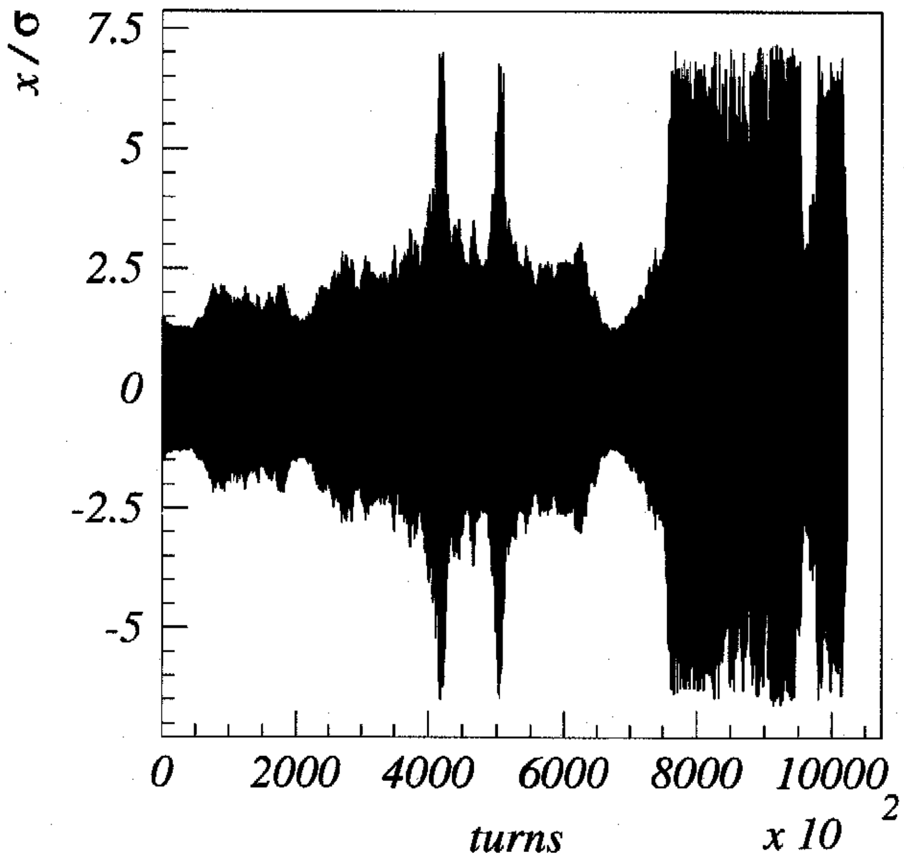
With tuneshift oscillation due to the synchrotron oscillation

initial particle x = 1.5 px = 0   qs = 0.001
lattice errors 20 sextupole a_2 = 0.01





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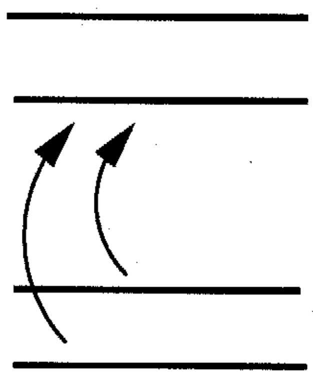
Conclusion:

1 Resonance induced diffusion leads a particle to the far resonance (from the beam center)



bare tune

3rd order resonance

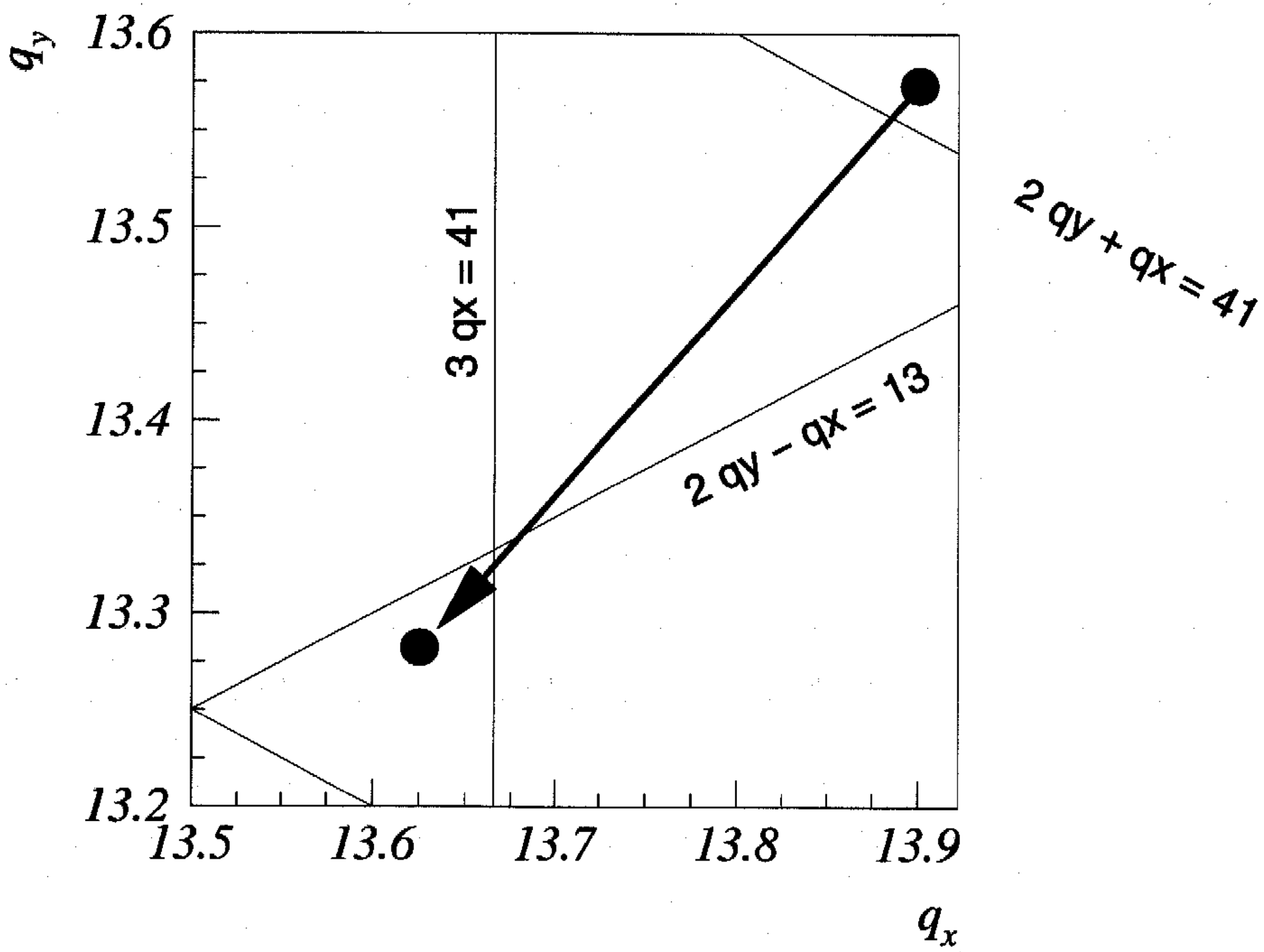
4th order resonance

beam center tune

2 the dangerous resonance are those which are outside the tunesperad !!!

In 4 dimension

Bare tunes qx = 13.91   qy = 13.58
Tuneshift   dqx = dqy = 0.3



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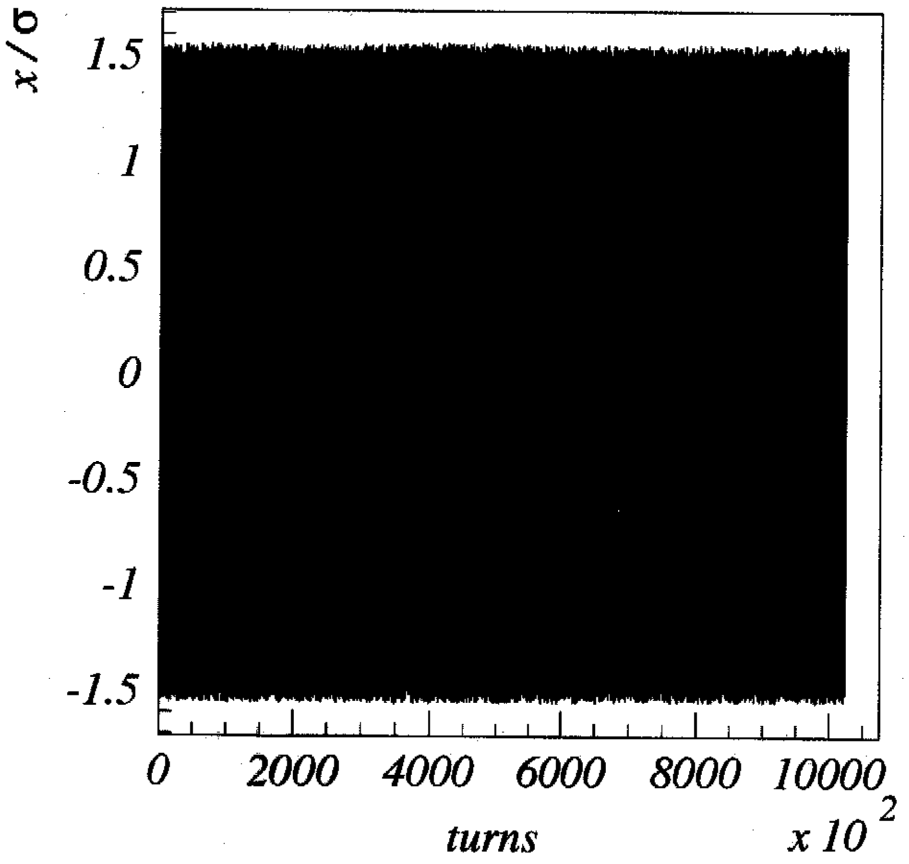
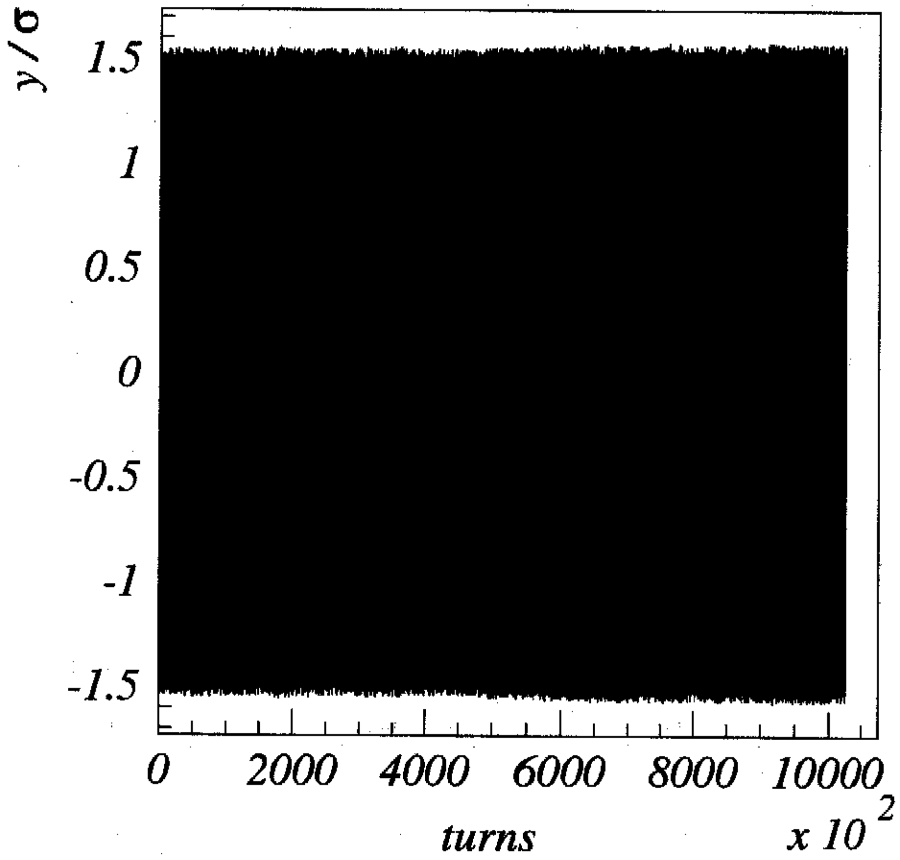
Here we excite the 40th harmonics. Long term effects are not visible





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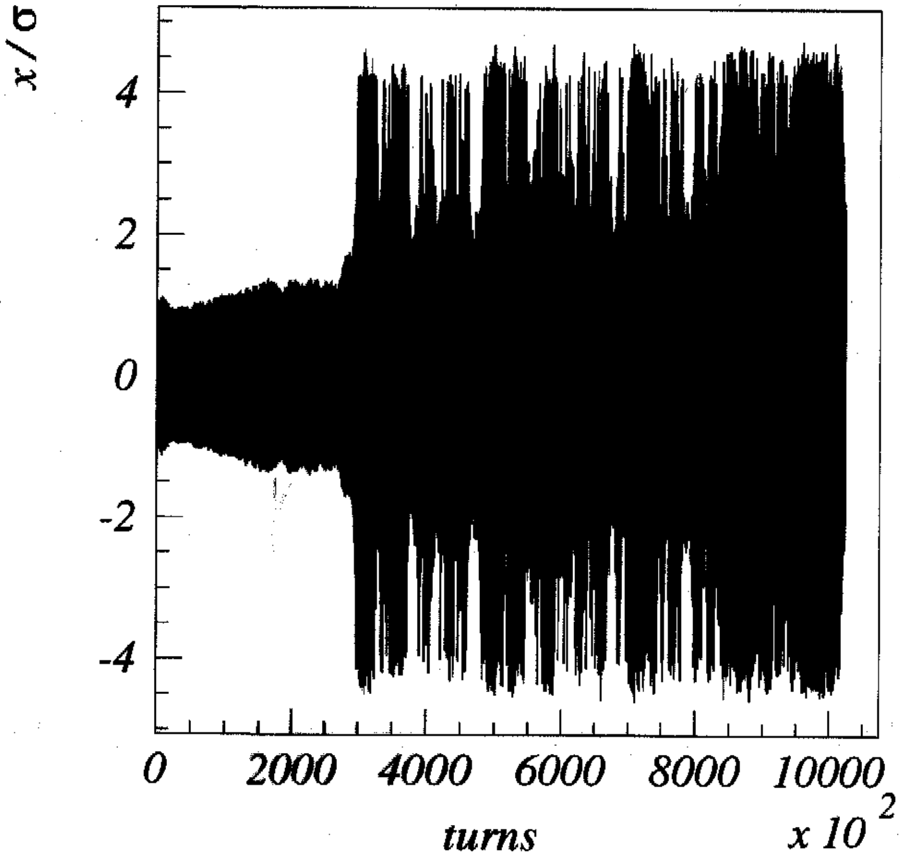
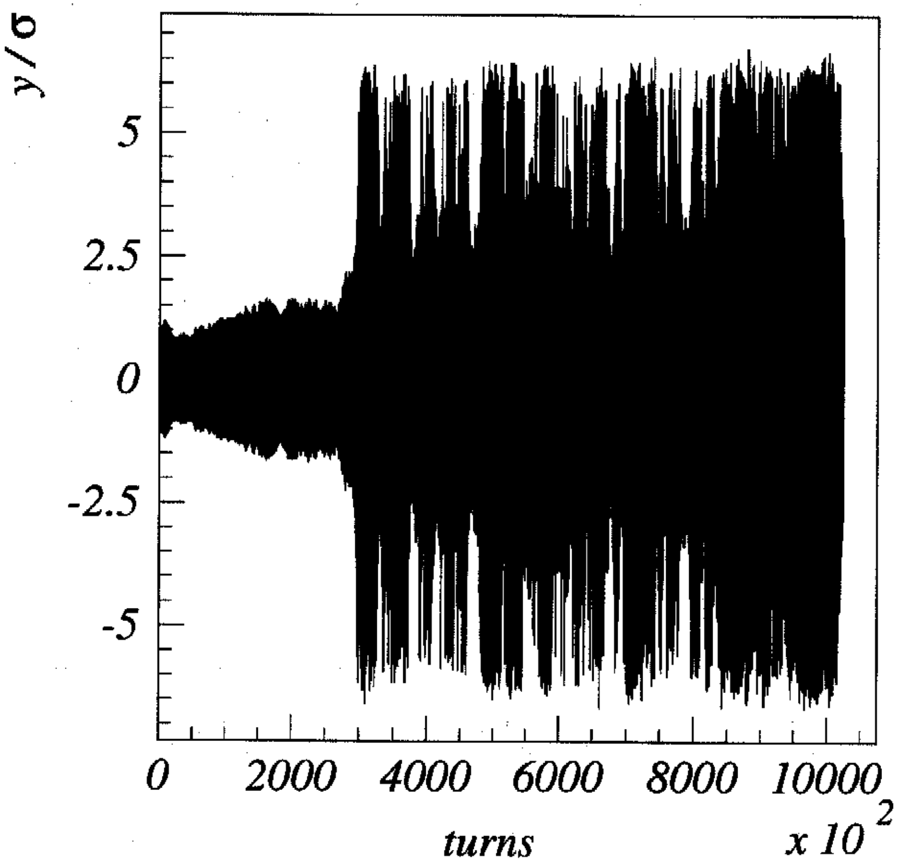
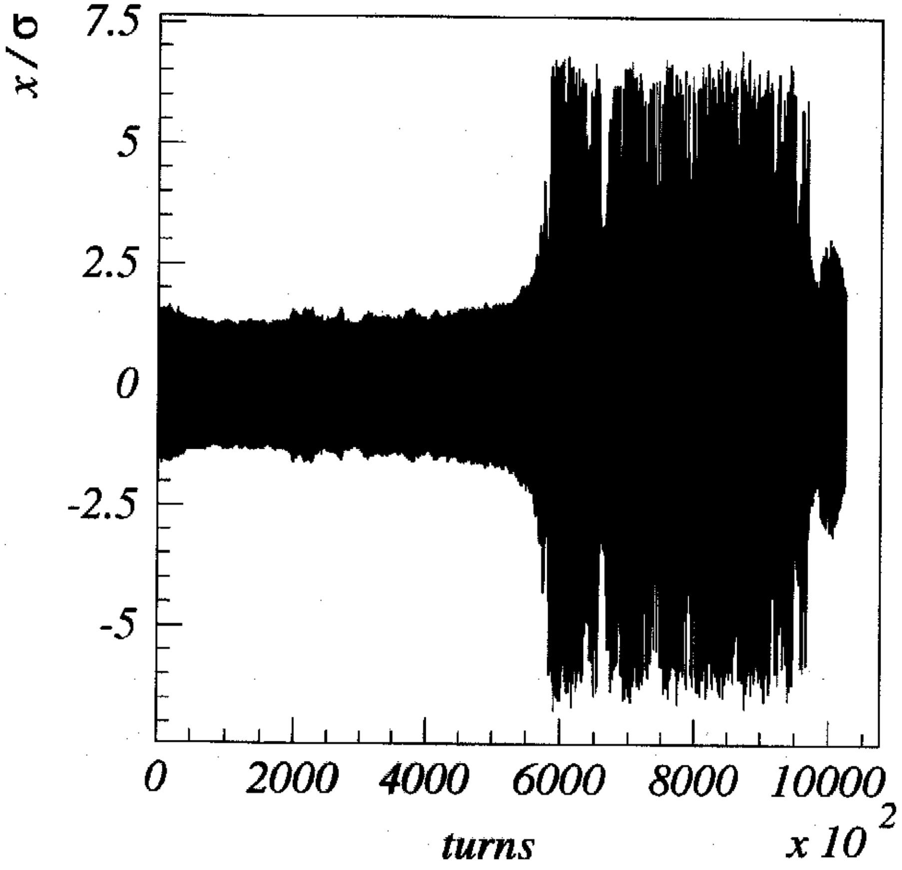
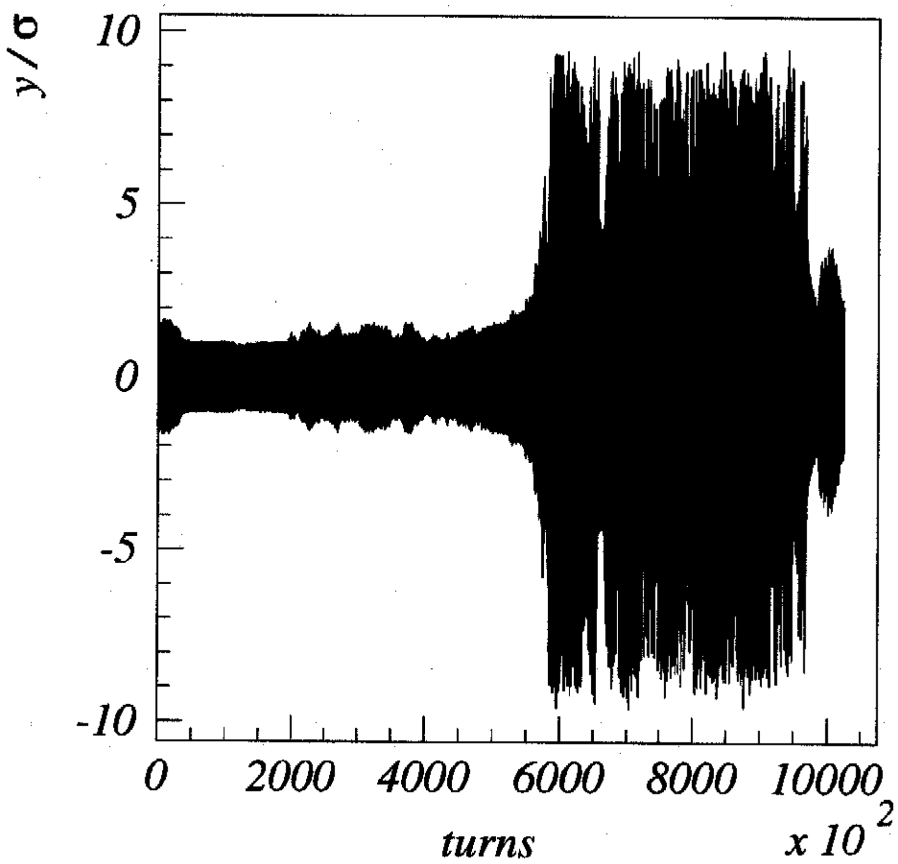
Sextupolar error excite the resonance lines 3 qx = 41, 2qy + qx = 41





Here the space charge is doubled dqx=dqy = 0.6
Sextupolar error excite the resonance line 2qy + qx = 41





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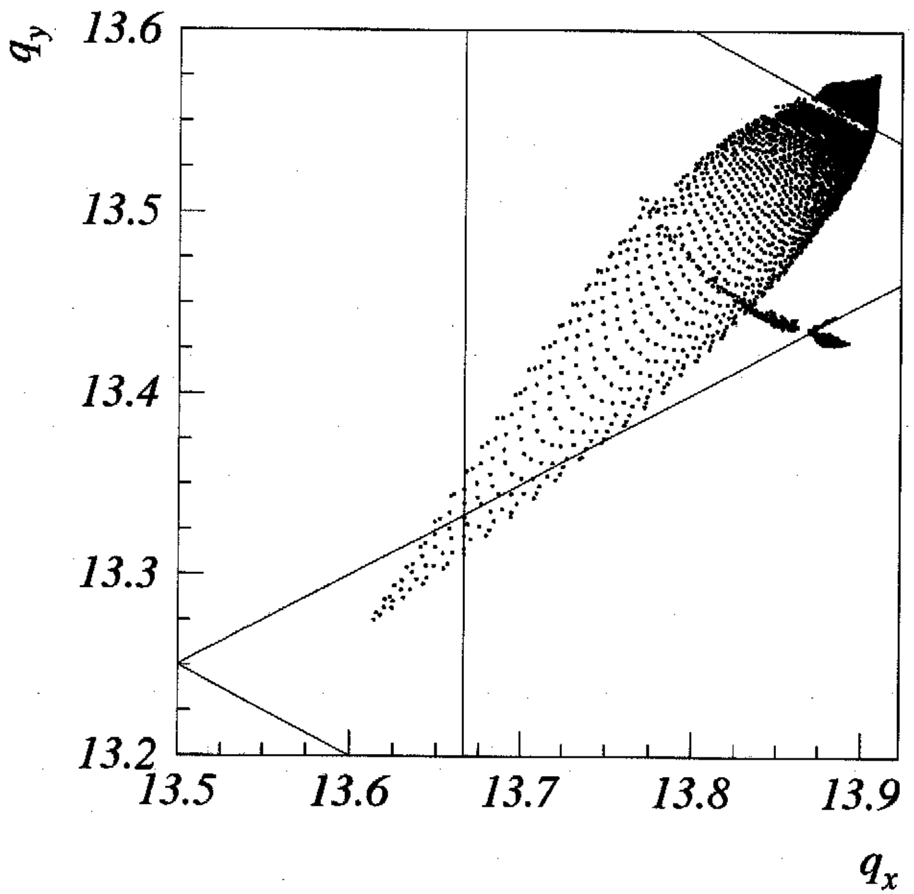
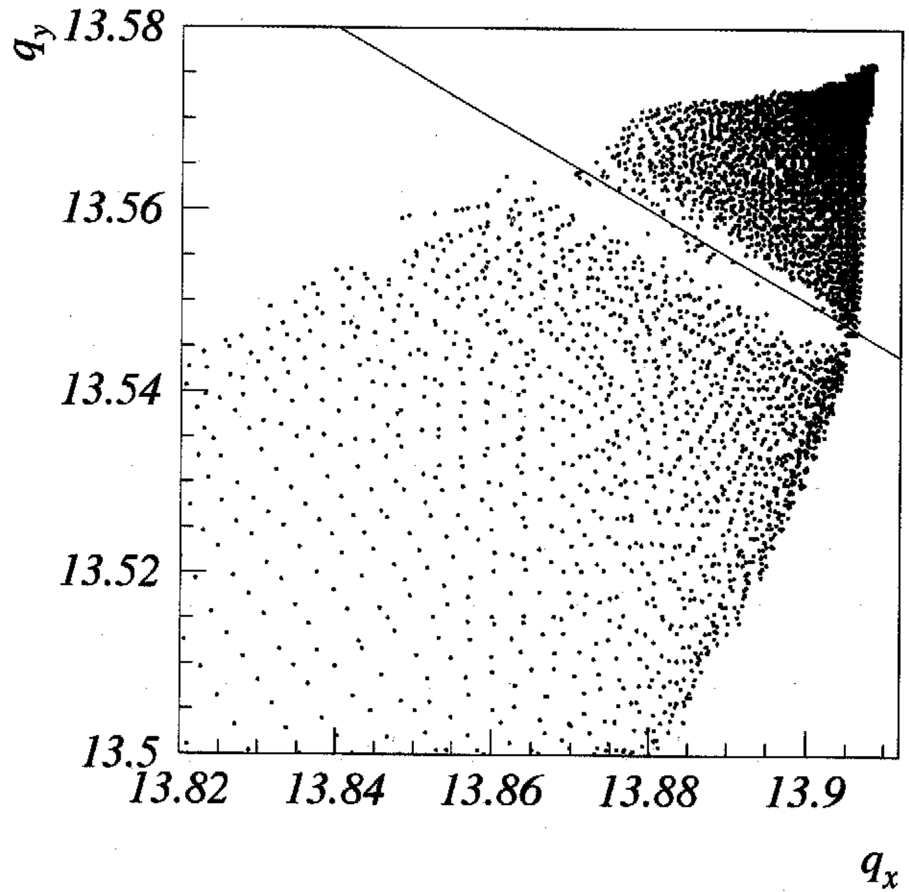
Tune footprint





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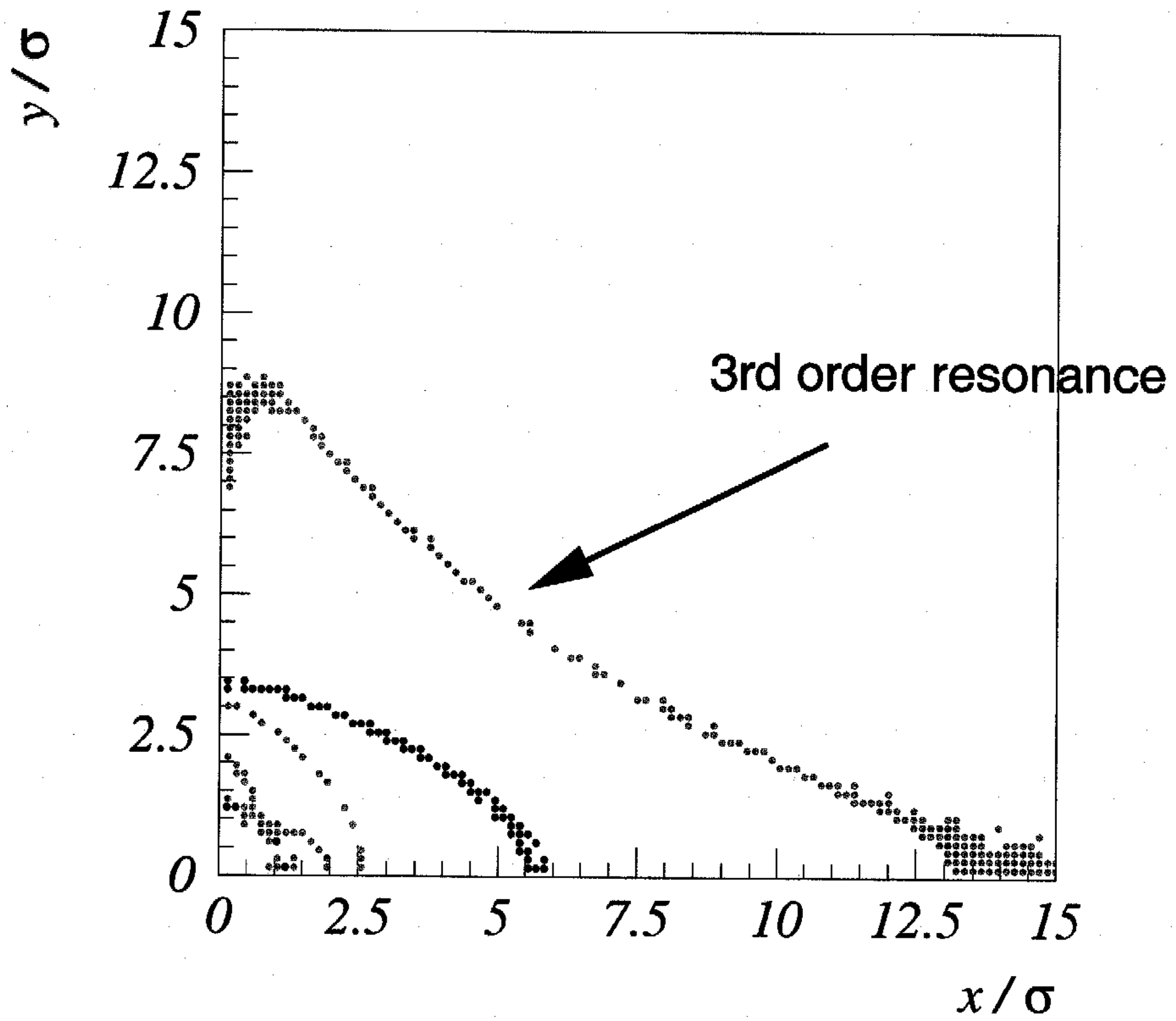
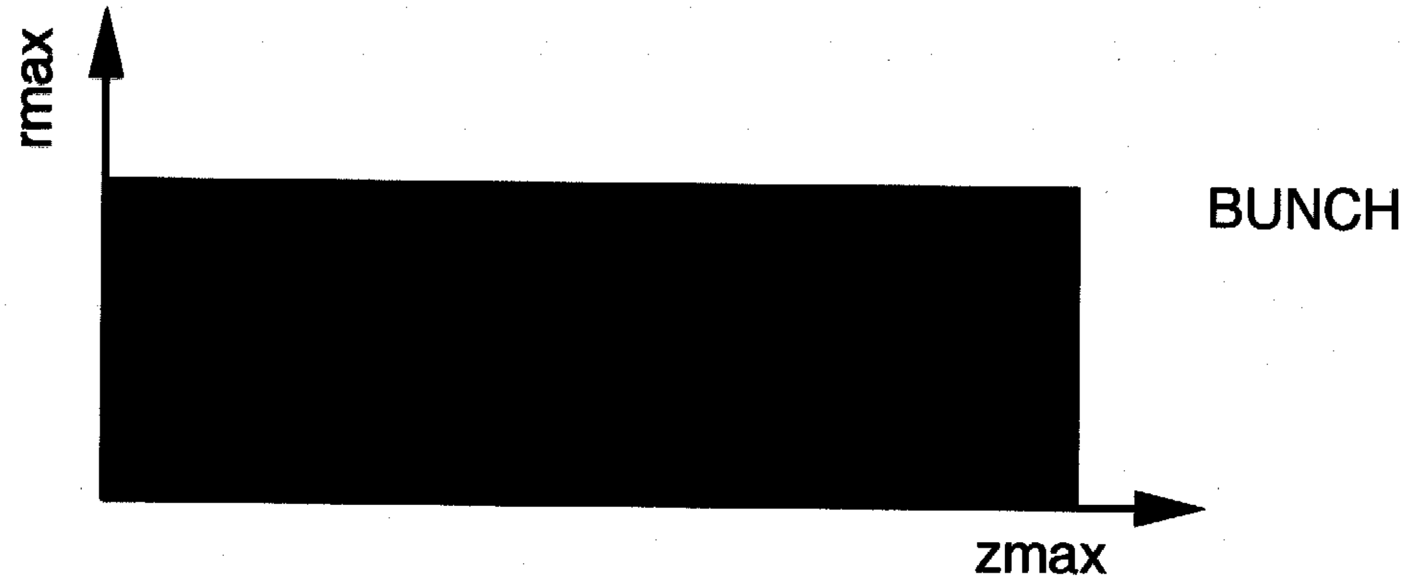
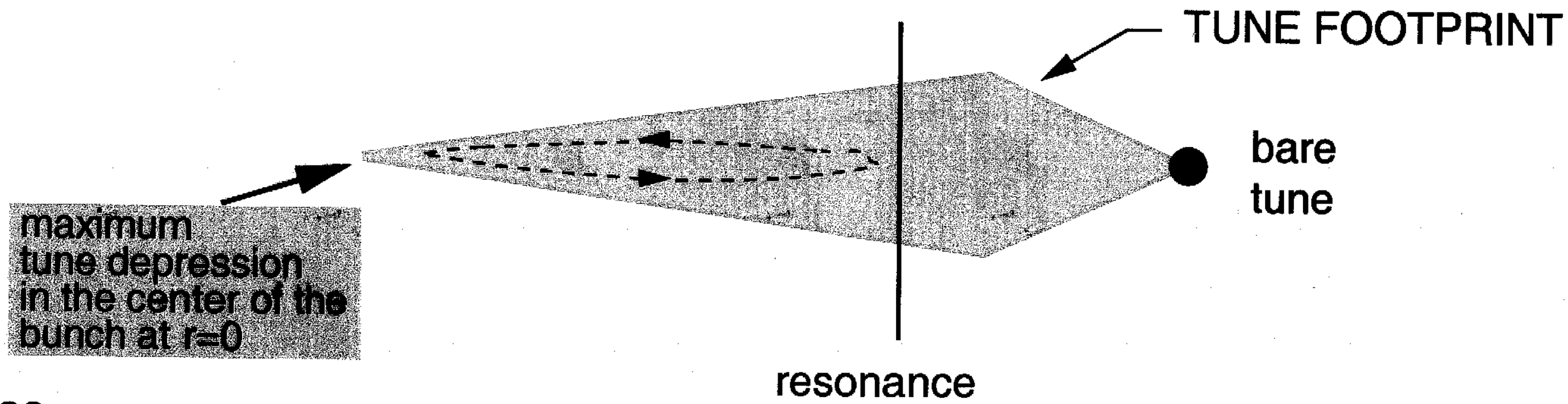
Position of the resonance



Single particle "tune dynamics" in a bunch





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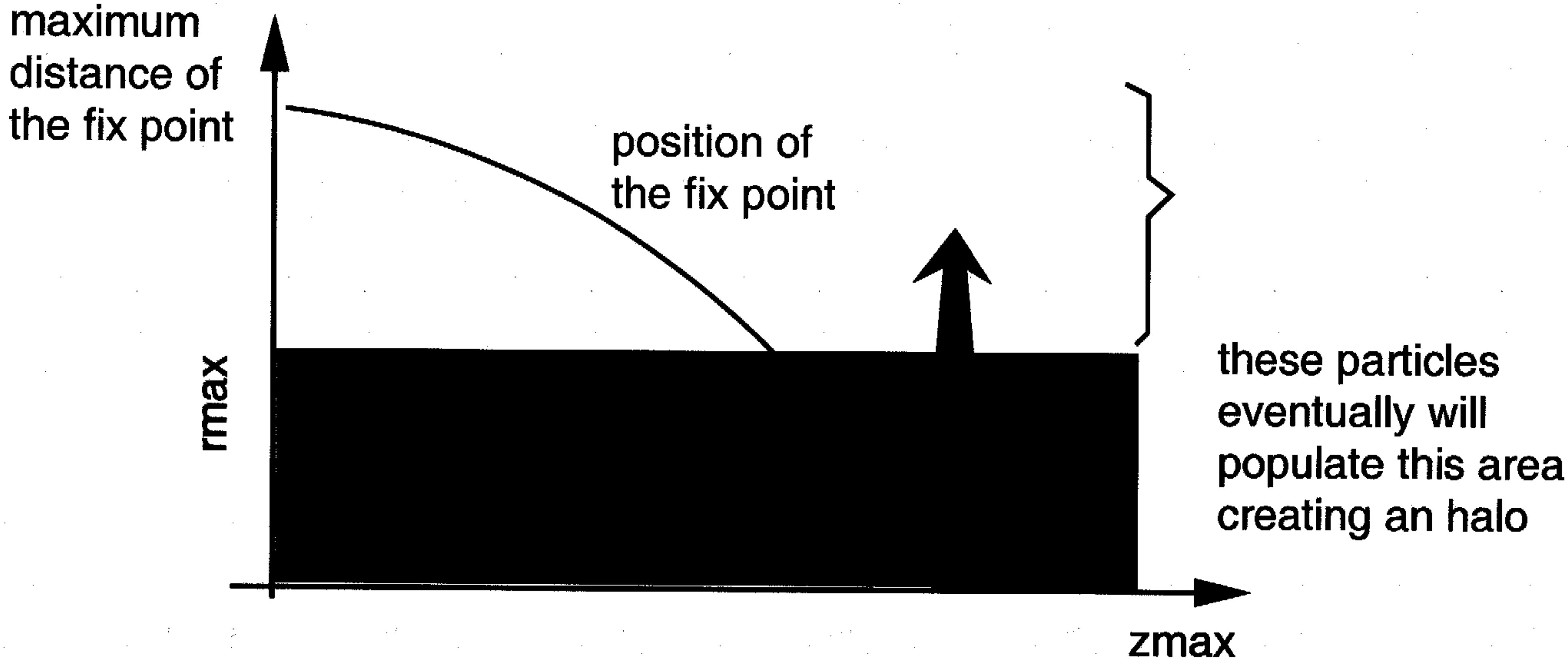

maximum
distance of
the fix point
position of
the fix point
rmax
these particles
eventually will
populate this area
creating an halo
zmax

Conclusion

when space charge is not to strong it effects the position of the resonances excited by lattice nonlinear errors

variation of space charge move resonances through phase space causing resonance – particle orbit crossing. We identified two effects

1 the resonance crossing causes a particle orbit bump

2 the resonance traps the particle.

periodic space charge modulation induced by the synchrotron motion may drive a particle to the position of the farest resonance crossed.

guideline for setting bare tunes in order to avoid long term loss