

Longitudinal Motion in Nearly-Isochronous FFAGs

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9 April 2002

Introduction



- Path length in FFAG arc not independent of energy.
 - ◆ Generally quadratic
 - ◆ Particle won't come back to RF at same phase each time
 - ◆ High-frequency RF: walk off crest
- Given the maximum path length deviation, how many turns can a particle be accelerated
 - ◆ Answer: infinite
 - ◆ Linac voltage goes to nonzero value as turns go to infinity
 - ◆ Minimum voltage increases linearly with path length deviation
- How does acceptance vary with number of turns?

Equations

- Equations of motion

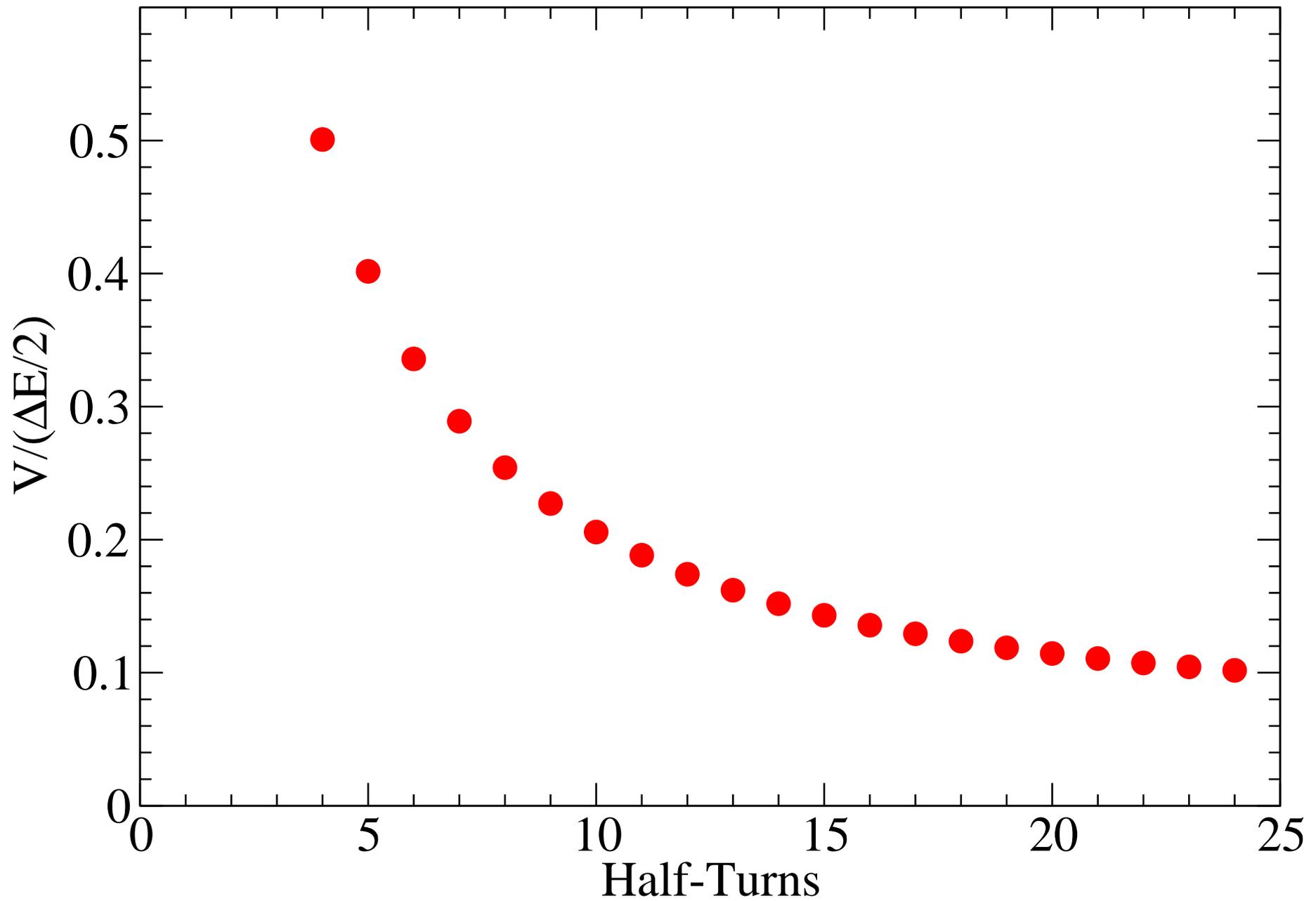
$$E_{n+1} = E_n + V \cos(\omega\tau_n) \quad \tau_{n+1} = \tau_n + \Delta T \left(\frac{E_{n+1} - E_{\text{avg}}}{\Delta E/2} \right)^2 - T_0$$

- Minimize V by varying τ_0, T_0 , with $E_N - E_0$ constrained to be ΔE
- As $N \rightarrow \infty$ (continuous approximation)

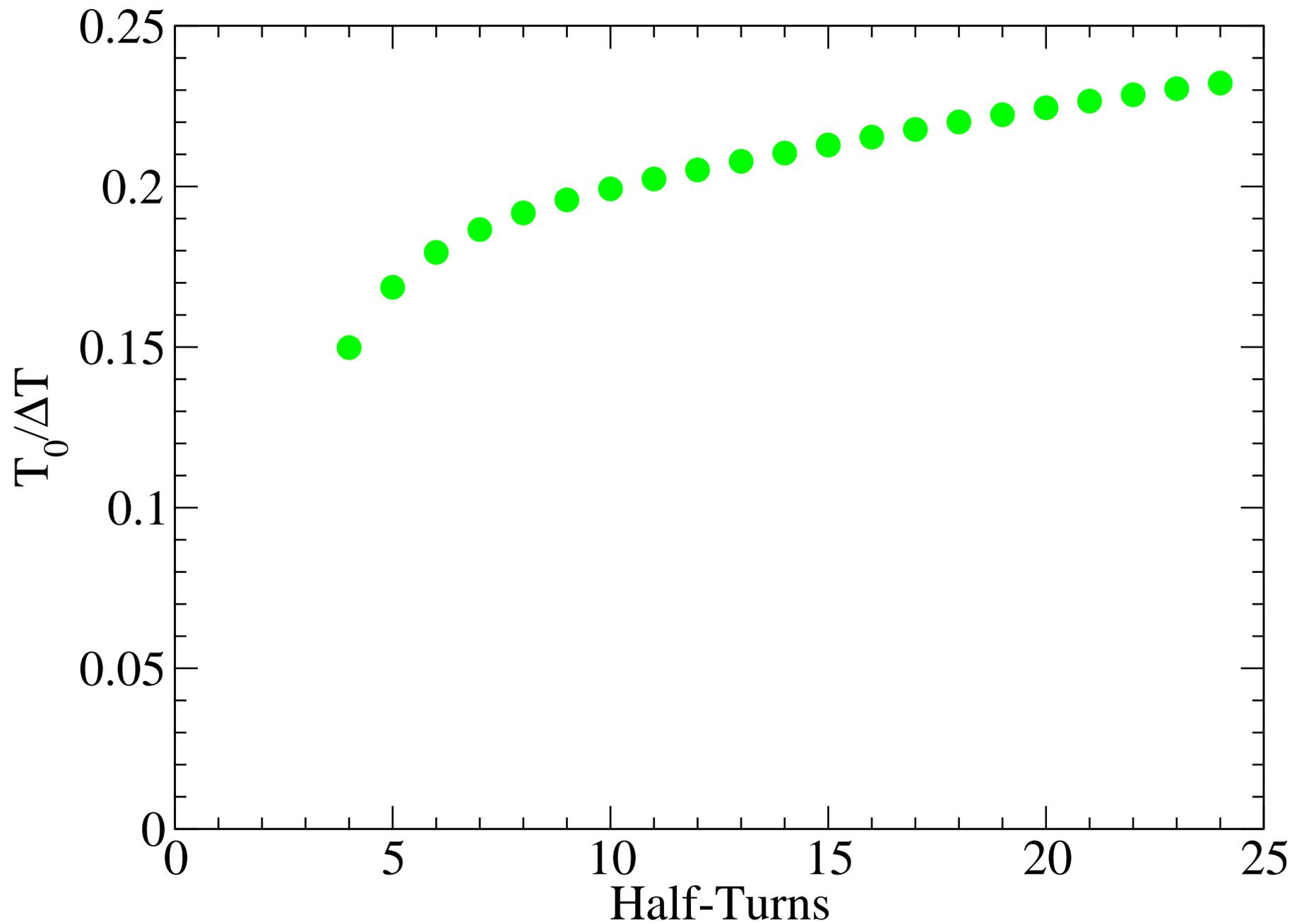
$$V \rightarrow \frac{\omega \Delta T \Delta E}{24} \quad T_0 \rightarrow \frac{\Delta T}{4} \quad \omega \tau_0 \rightarrow \frac{\pi}{2}$$

- Motion: crossing crest three times

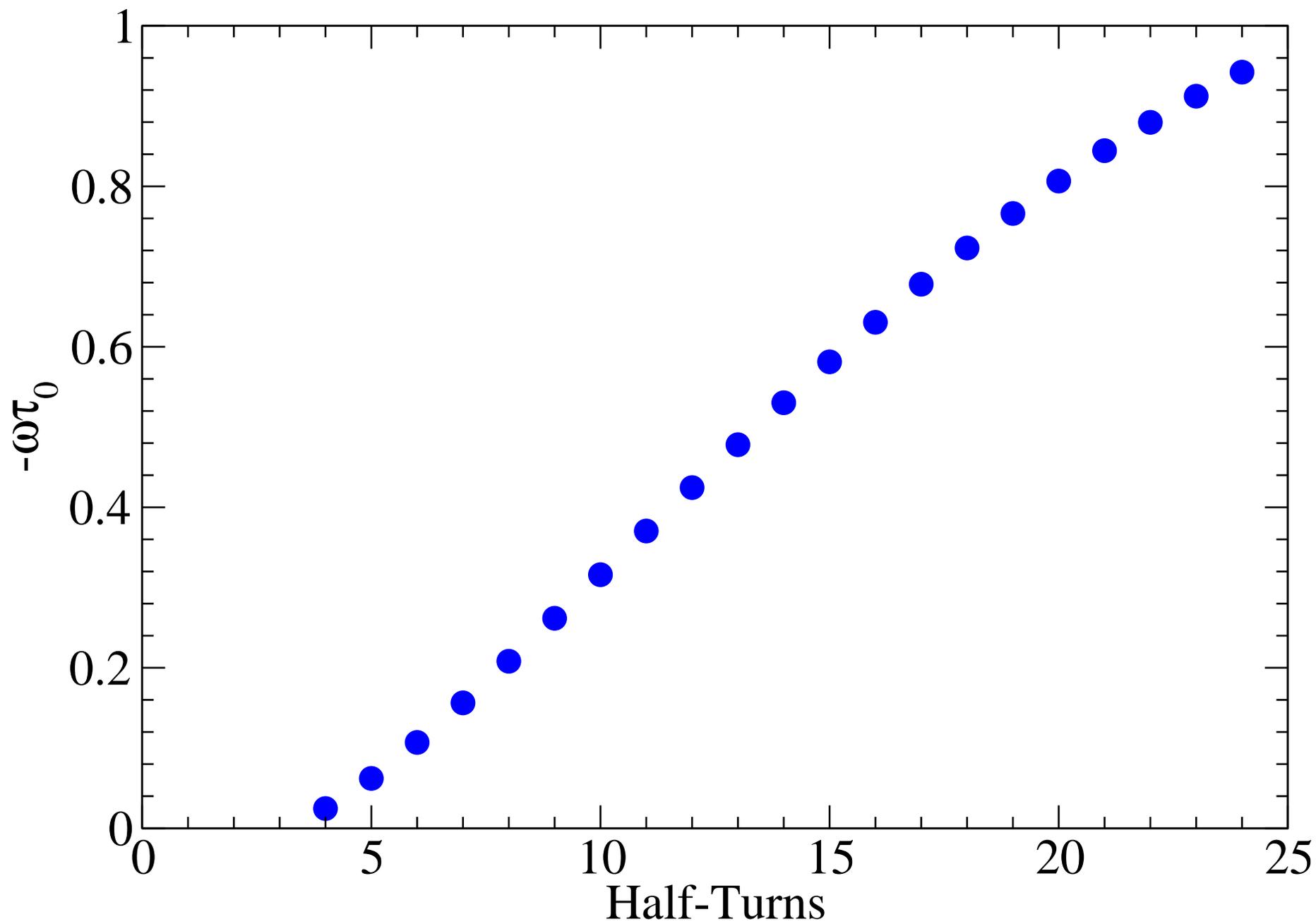
Linac Voltage

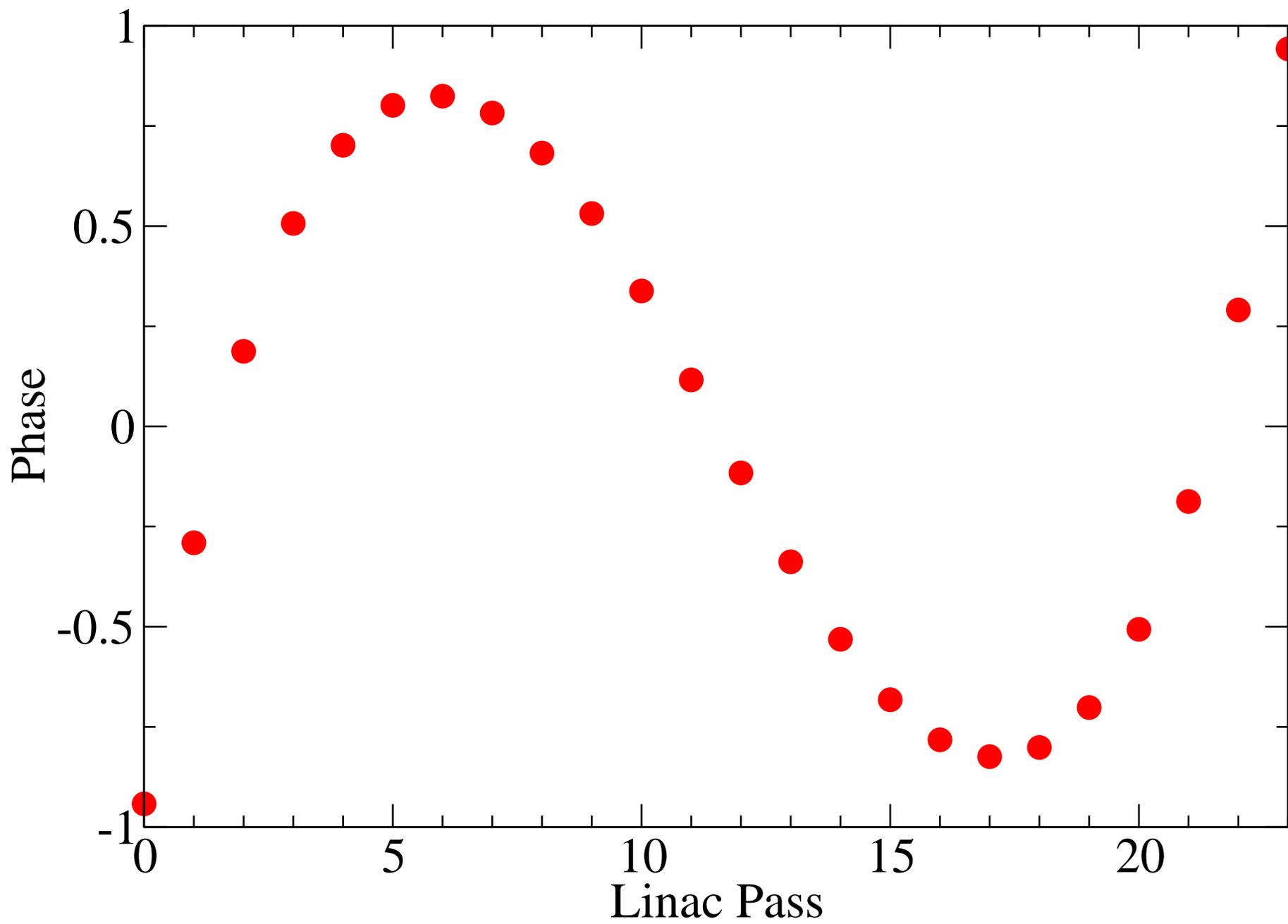


Path Length Offset



Initial Phase





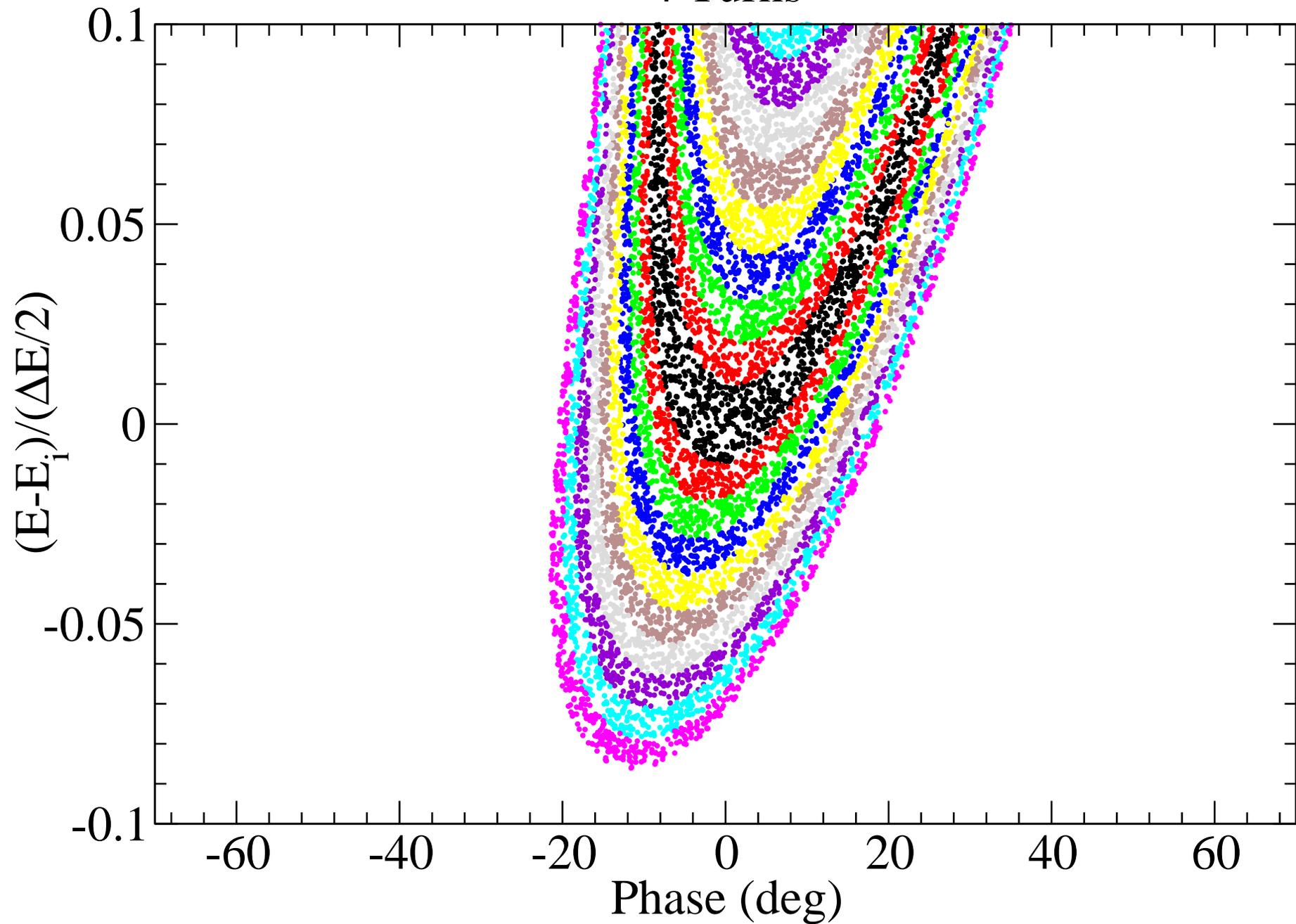
Observations



- Linac costs won't decrease too rapidly with more turns
- Phase space acceptance
 - ◆ Energy acceptance reduced with more turns
 - ◆ Time acceptance may increase slightly?

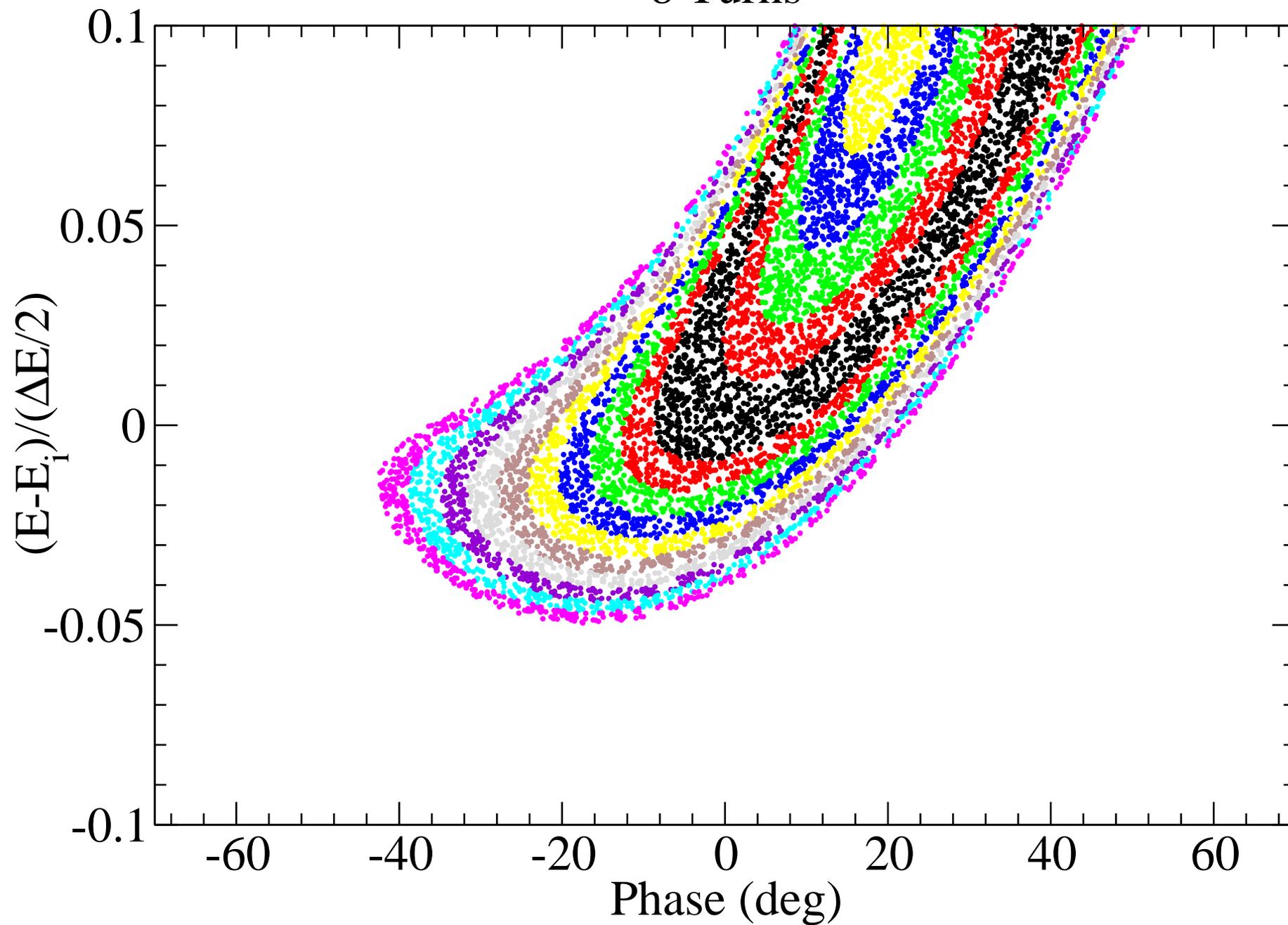
Initial Phase Space

4 Turns



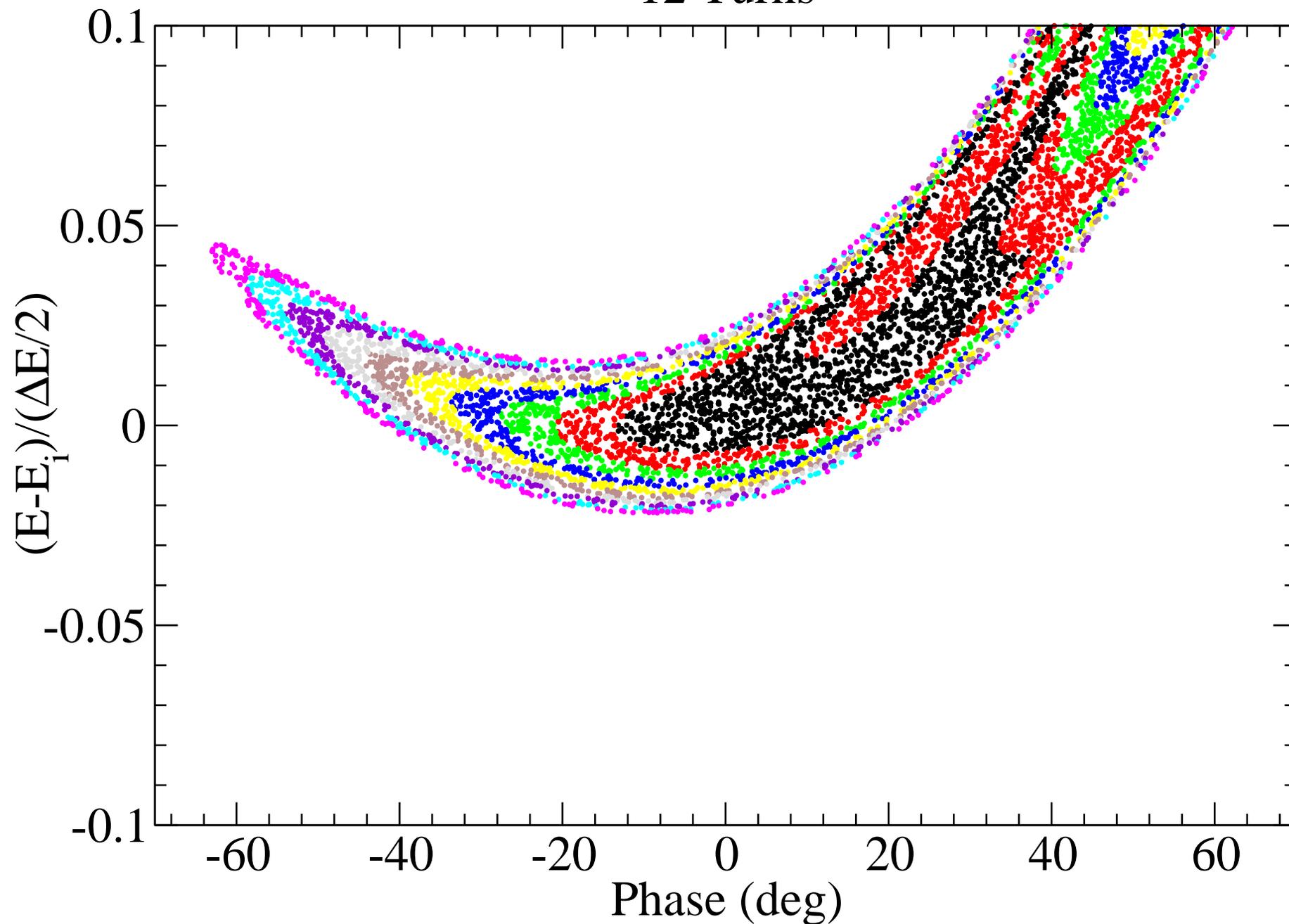
Initial Phase Space

8 Turns



Initial Phase Space

12 Turns



Potential Improvements



- Allow linacs to have different phases, arcs different time offsets
- Increase voltage above optimum: more phase space acceptance?
- Effect of beam loading