Radio-frequency schemes for acceleration of muons in a non-scaling FFAG

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FFAG has, in principle, large transverse and longitudinal acceptances. But…

Orbit length versus momentum

$$\frac{\Delta C}{C} = \alpha \frac{\Delta p}{p} + \beta \left(\frac{\Delta p}{p} \right)^2 + \cdots$$

	Synchrotron	**Non-scaling FFAG**
	$\beta << \alpha$	$\beta >> \alpha$
	$\frac{\Delta p}{p} << 1\%$	$\frac{\Delta p}{p} < 10\%$

Path-length changes ⇒ cavity must be re-phased to maintain synchronism

⇒ acceleration must be slow enough compared with cavity filling time:

$$\left. \frac{\Delta p}{p} \right|_{turn} < \frac{h}{Q} = \frac{\text{harmonic number}}{\text{quality factor}}$$

Muons are a special case: to avoid decay losses, acceleration must be extremely rapid e.g. 6 to 20 GeV in 5 turns

Ideally need giga-volts per turn and frequency agility $Q<10^3$.

	Low power	Frequency agile
NC	✗	✓
SC	✓	✗

E.g. 200MHz, $Q=7\times10^4$, R=14MΩ, 1800 cavities, power consumption is 360MW. Instead choose super-conducting cavity.

What can be done with fixed radio-frequency?

If no special measures, then acceleration is stochastic and transmission to extraction energy → zero

Optimization algorithm

Assume reference particle at centre of bunch at centre of train receives the ideal energy gain at each cavity crossing, and record the precise arrival times at each cavity each turn.

Ideally phase is zero at each traversal [Cosine(0)=1] so that full voltage is obtained.

Guess a frequency f

At ideal arrival times t_{ij} calculate phases of each cavity $\phi_{ij}=f\times t_{ij}$

$$S = \sum_{j}^{turns} \sum_{i}^{cavity} (\phi_{ij} - \overline{\phi_{ij}})^2$$

Is S a minimum?

N (return to: Guess a frequency f)

Y

Set $f=f_{opt}$

Set initial cavity phases $= -\phi_{ij}$ at $t=0$

then $\phi_{ij}\approx 0$ at t_{ij}

This procedure alone is not sufficient to guarantee good transmission (even for the reference particle!)

Over-voltage factor

Since the time of Veskler & McMillan voltages over the nominal have been employed. Acceleration off the crest of the waveform at ϕ_s so that $V_0 = \hat{V} \times \mathrm{Cos}\,\phi_s$.

Trick: place 1 particle at the centre of each bunch in the train. Because the optimal frequency is not equal to the injection frequency, there is a spread of injection timing errors along the train.

Choose $\hat{V}$ numerically to minimize

$$\sum_{n}^{bunches} (E_n - E_{ext}^0)^2$$

to make acceleration more tolerant of errors.

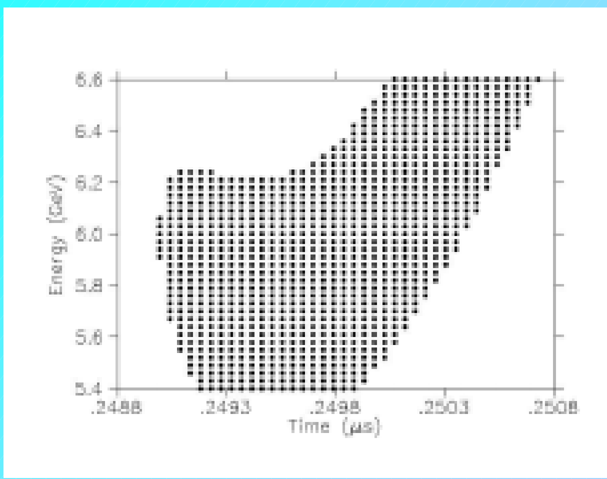
The combination of optimal frequency, best initial phases and an over-voltage leads to remarkably good transmission, but cannot alter the fact that the dynamics is strongly non-linear (even when cavities are re-phased to ideal values each turn). Moreover the length of the bunch train is limited by phase slip and leads to energy and emittance variation.

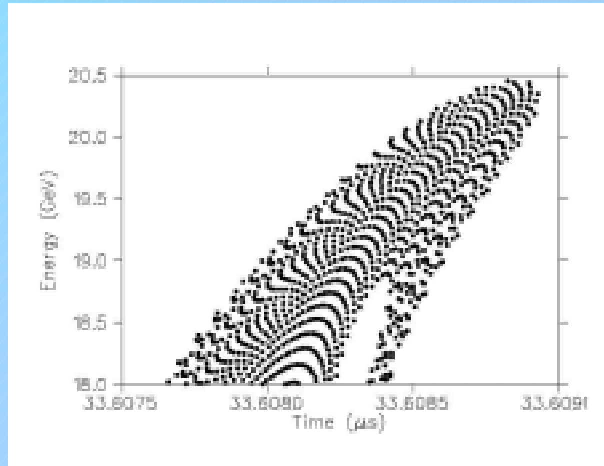
To map out the input acceptance and output emittance, flood 360 deg of rf phase with a ±10% energy band at injection and track to extraction energy or until lost. Record the survivor particles. Repeat for all bunches in the train to find energy and emittance variation from head to tail of train.

Apply to a 2 km circumference 6-20 GeV FFAG with 300 lattice cells, and 200 MHz RF.

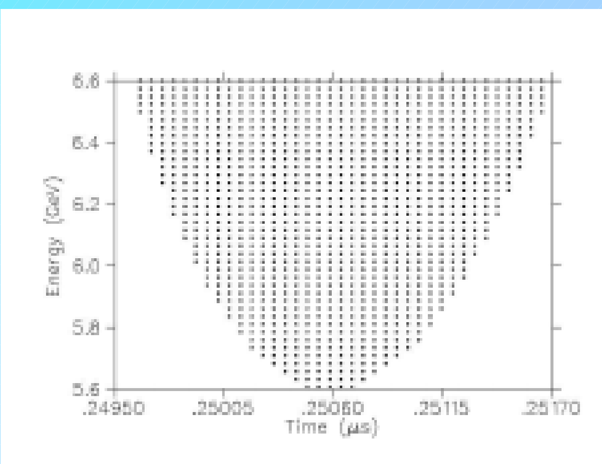
Comparison of transmission for simple rf schemes

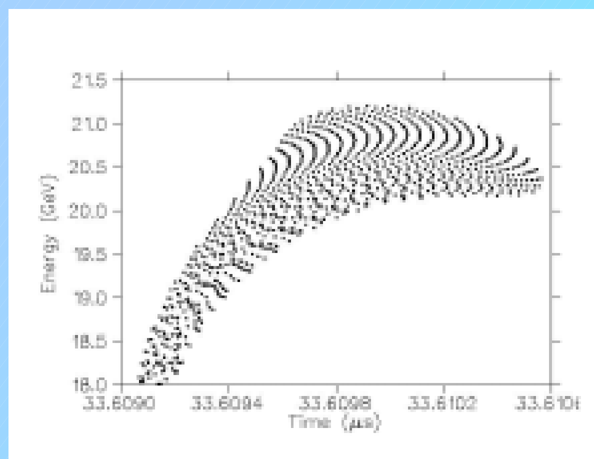
Ideal re-phasing, no over-voltage, 5 turns, 1.2 eV.s



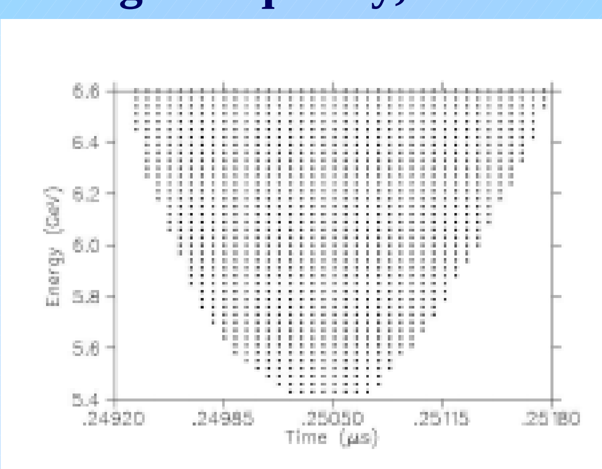


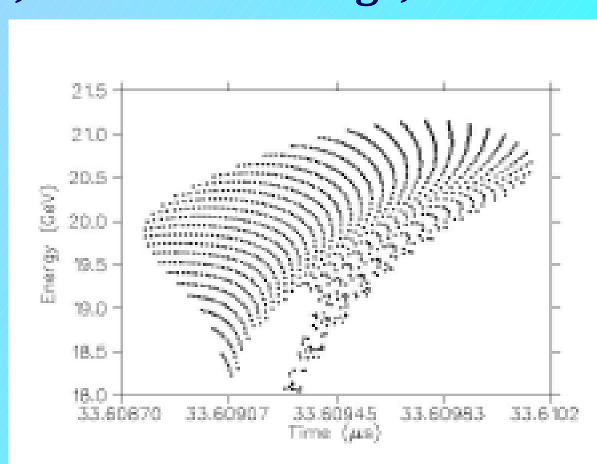
Single frequency, 40% over-voltage, 5-turns, 1.37 eV.s





Single frequency, dual harmonic, 25% over-voltage, 1.9eV.s





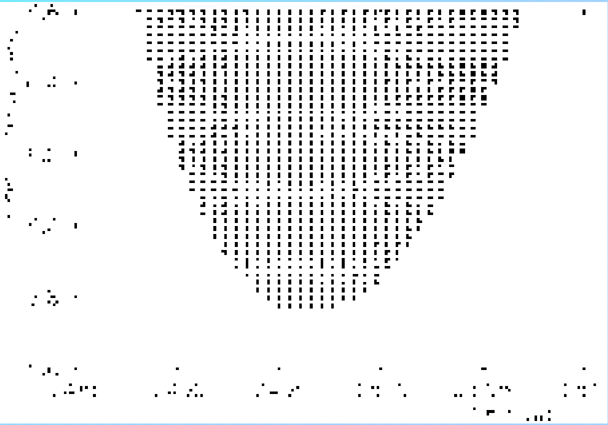
Multiple frequencies

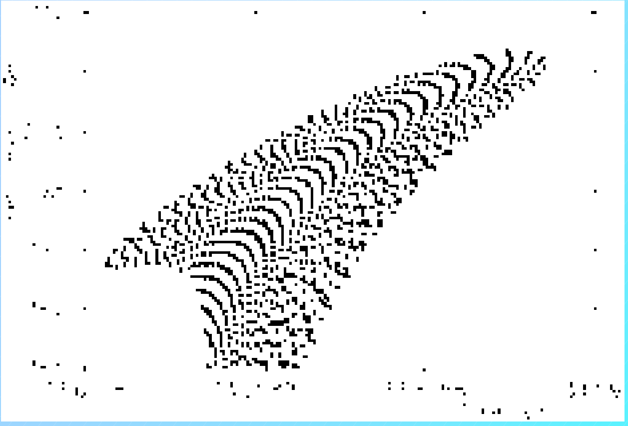
If there are correlations between cavities over several turns, one may reduce the sum of phase deviations by arranging the cavities in groups each with its own frequency.

We have considered groupings of the form ABCDABCDABCDABCD and AAAABBBBCCCCDDDD

The number of groups is equal to the number of frequencies to be found, and may be an integer factor of the total number of cavity cells. Increasing turns lowers electric field gradient.

Single frequency, 25% over-voltage, 5-turns, 1.0 eV.s





300 frequencies, 25% over-voltage, 6-turns, 0.93 eV.s