Simulation Study of Beam-beam Effects in Run IIa at Collision

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- **Introduction to Tevatron RUN II, Beam-beam study in RUN IIa(36X36)**
- **Simulation model for Beam-beam interactions(headon and long-range)**
- **Footprints and dynamic aperture(DA) calculations**
  - **Single beam**
  - **All beam-beam interactions for Bunch 6**
  - **All beam-beam interactions for Bunch 1 and 12**
- **Resonance scan**
- **Beam-beam study with crossing angles**
- **Conclusion**

Introduction to Tevatron RUN II

- Multiple IC makes the event reconstruction and physics analysis more difficult.
  - CDF(B0) would prefer no more than about 3 to 4 IC
  - D0 would prefer no more than 1 to 2 IC
- The limit on the number of Interactions per Crossing(IC) + more luminosity pushes us to more buntches.

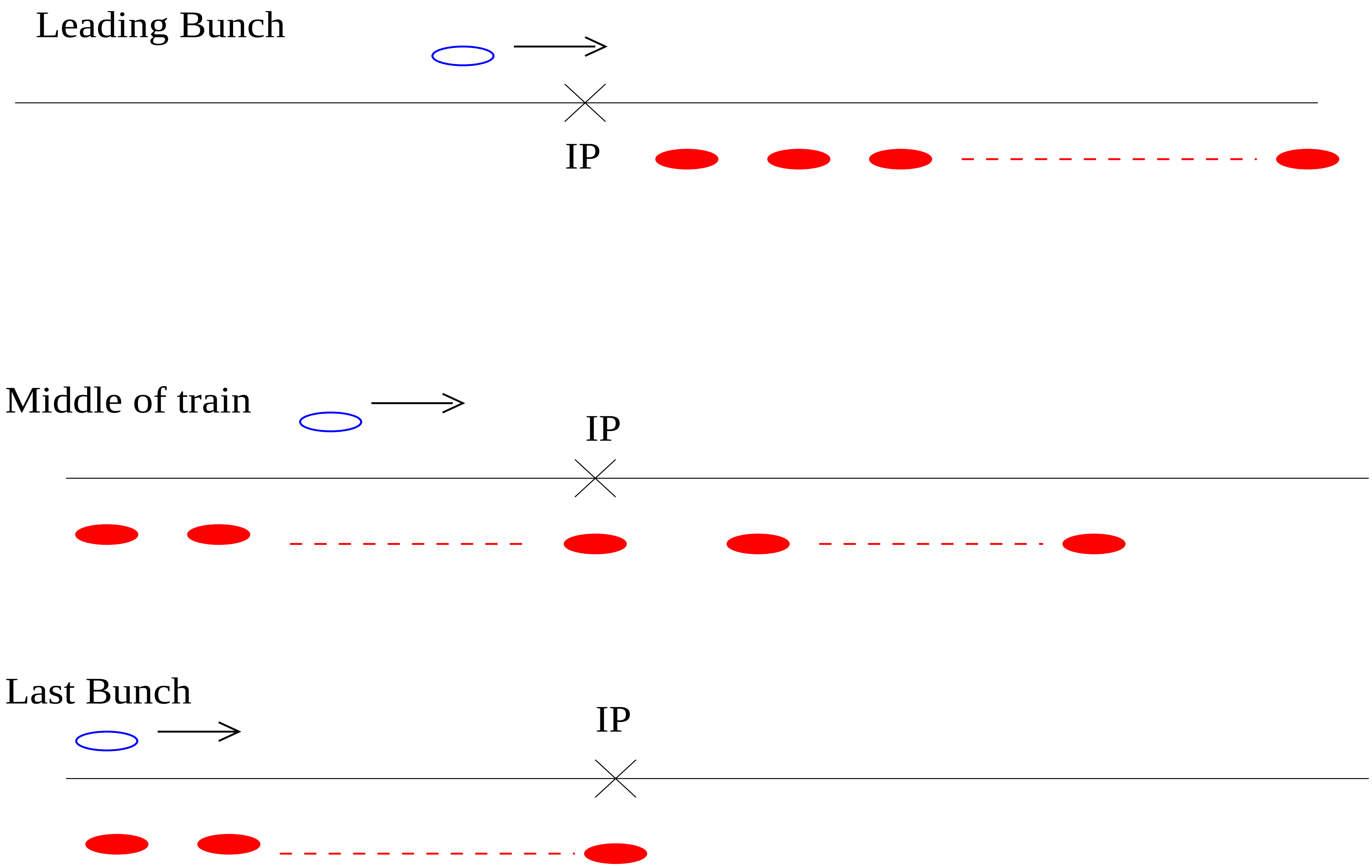
Table 1: Main beam parameters in Run I and Run II

	Run Ib $p/\bar{p}$	Run IIa $p/\bar{p}$	Run IIb $p/\bar{p}$
Luminosity [$cm^{-2}sec^{-1}$]	1.6×10^{31}	8.6×10^{31}	14.0×10^{31}
Interactions per crossing	4.9	2.3	1.4
Energy [GeV]	900	1000	1000
Bunch Intensities $\times10^{11}$	(2.3/0.55)	(2.7/0.3)	(2.7/0.3)
Emittances 95% [mm-mrad]	23/13	20/15	20/15
Number of bunches(pX$\bar{p}$)	6X6	36X36	140X103
Bunch separation [m]	1049.3	118.8	39.6
Beam size at IP [μm]	37/28	33/29	33/29
β^* [cm]	35	35	35
Longitudinal emittance [eV-sec]	3.5	2.0	2.0
Bunch length [cm]	49	37	37
Half crossing angle in 45^o plane[μ rad]	0	0	192
Beam-beam parameter/IP $\times10^{-3}$	3.4/7.4	1.5/9.9	0.385/3.0

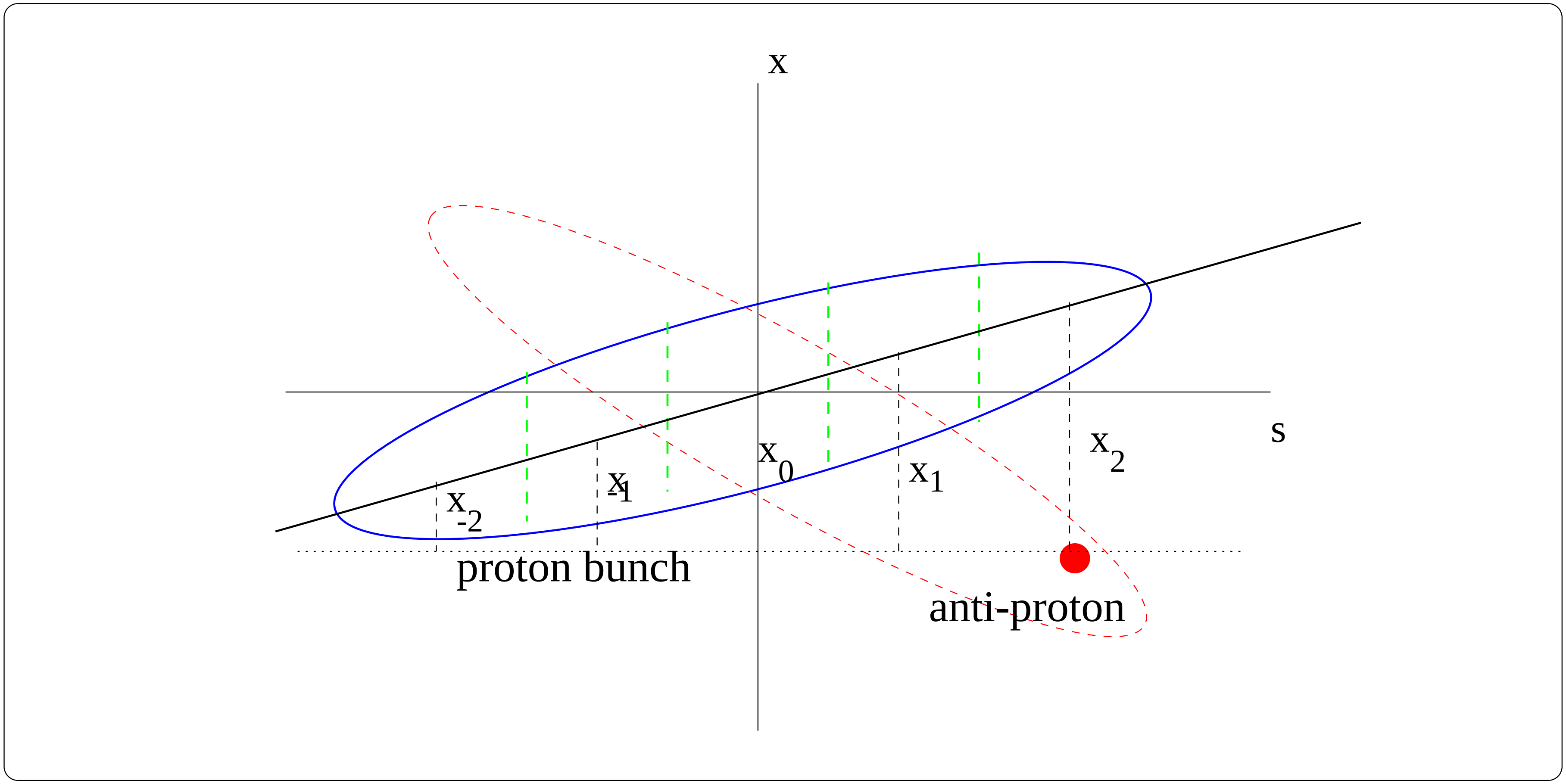
Beam-beam Study in Tevatron RUN IIa(36X36)

- Higher Design proton intensities per bunch $\Longrightarrow$ higher the headon beam-beam tune shifts expierenced by the anti-protons
- Long-range interactions $\Longrightarrow$ More total beam-beam induced tune spread of the anti-proton
- Beam-beam effects + the nonlinear fields in the IR quadrupoles + the chromaticity sextupoles $\Longrightarrow$ Smaller the Dynamc Aperture(DA)/ the lifetime of the anti-protons significantly
- The impact on DA due to Crossing angle for study purpose

Bunch to bunch collisions



Beam-beam simulation model for headons interaction



The strong proton bunch is divided into 9 slices

At the Tevatron where $\sigma_s \simeq 36$cm, $\beta^* = 35$cm

Bunch length effects

Hourglass Effects: The transverse size of each disk is different because of the change in β function.

For example the beta function and rms size in the horizontal plane is

$$\beta_x(i) = \beta_x^* + \frac{l_i^2}{\beta_x^*} = \beta_x^* + \frac{(s_i)^2}{\beta_x^*} \tag{1}$$

$$\sigma_x(i) = \sqrt{[1 + (\frac{(s_i)}{\beta_x^*})^2]}\sigma_x^* \,. \tag{2}$$

The luminosity reduction for round beams is given

$$\mathcal{R}(u) = \frac{\mathcal{L}}{\mathcal{L}_0} = \sqrt{\pi}u \exp[u^2]\mathrm{erfc}(u) \tag{3}$$

where $u = \beta^*/\sigma_s$. At the Tevatron where $\sigma_s \simeq 36$cm, $\beta^* = 35$cm, resulting in the reduction about 25%.

Longitudinal density variation: a Gaussian distribution

If $\rho(s_i)$ is the density of the i'th slice, the total charge in the bunch with N_s slices is

$$Q = \sum_i^{N_s} \Delta q_i = e \sum_i^{N_s} \rho(s_i)\Delta s_i \tag{4}$$

where Δs_i is the thickness of the i'th slice. If we assume that all slices are of uniform thickness $\Delta s_i = 2\sigma_s/N_s$, the fraction of the total charge in each slice is

$$\frac{\Delta q_i}{Q} = \frac{1}{\sum_{i=1}^{N_S} \exp[-i^2/(2N_S^2)]} \exp[-\frac{i^2}{2N_S^2}] \tag{5}$$

We assume that the effective length of the bunch is $2\sigma_s$.

Phase Variation: Propagation between slices

From the $i-1$'th slice to the i'th slice, each of thickness Δs, the drift map is in the horizontal plane:

$$M_d(s_{i-1} \to s_i) = \begin{pmatrix} 1 & \frac{1}{2}\Delta s \\ 0 & 1 \end{pmatrix} \tag{6}$$

and similarly in the vertical plane.

Beam-beam kick

The kick on an anti-proton from the i'th slice of charge in the opposing proton bunch is

$$\Delta x' = 8\pi\xi_i \frac{x - x_i}{[(x-x_i)^2 + (y-y_i)^2]} \left\{1 - \exp[-\frac{[(x-x_i)^2]}{\sigma_x(i)^2} + \frac{[(y-y_i)^2]}{2\sigma_y(i)^2}]\right\} \tag{7}$$

$$\Delta y' = 8\pi\xi_i \frac{y - y_i}{[(x-x_i)^2 + (y-y_i)^2]} \left\{1 - \exp[-\frac{[(x-x_i)^2]}{\sigma_x(i)^2} + \frac{[(y-y_i)^2]}{2\sigma_y(i)^2}]\right\} \tag{8}$$

where ξ_i, the beam-beam strength parameter for the slice, is

$$\xi_i = \xi \frac{\Delta q_i}{Q} \tag{9}$$

In the expression for the kicks, x is the horizontal distance from the longitudinal axis which goes through the center of the detector and x_i is the corresponding distance of the i'th slice.

Complete beam-beam map

The map across the strong bunch is of the form

$$\mathcal{M} = \mathcal{M}_{BB}(N_S) M_d(N_S - 1 \to N_S) \mathcal{M}_{BB}(N_S - 1) M_d(2 \to 3) \ldots \mathcal{M}_{BB}(2) M_d(1 \to 2) \mathcal{M}_{BB}(1) \tag{10}$$

where we start with a beam-beam kick from the 1st slice, followed by a drift through the slice of thickness $2\sigma_s/N_S$, followed by a kick etc. and ending with a beam-beam kick from the last slice N_S.

Beam-beam model for Long-range interactions - δ function

The kicks are given by the Bassetti-Erskine expressions for Gaussian beams(assuming $\sigma_x > \sigma_y$)

$$\Delta x' = \frac{N_b r_p}{\gamma_p}\sqrt{\frac{2\pi}{\sigma_x^2 - \sigma_y^2}}\mathrm{Im}\ F(x,y) \tag{11}$$

$$\Delta y' = \frac{N_b r_p}{\gamma_p}\sqrt{\frac{2\pi}{\sigma_x^2 - \sigma_y^2}}\mathrm{Re}\ F(x,y) \tag{12}$$

and

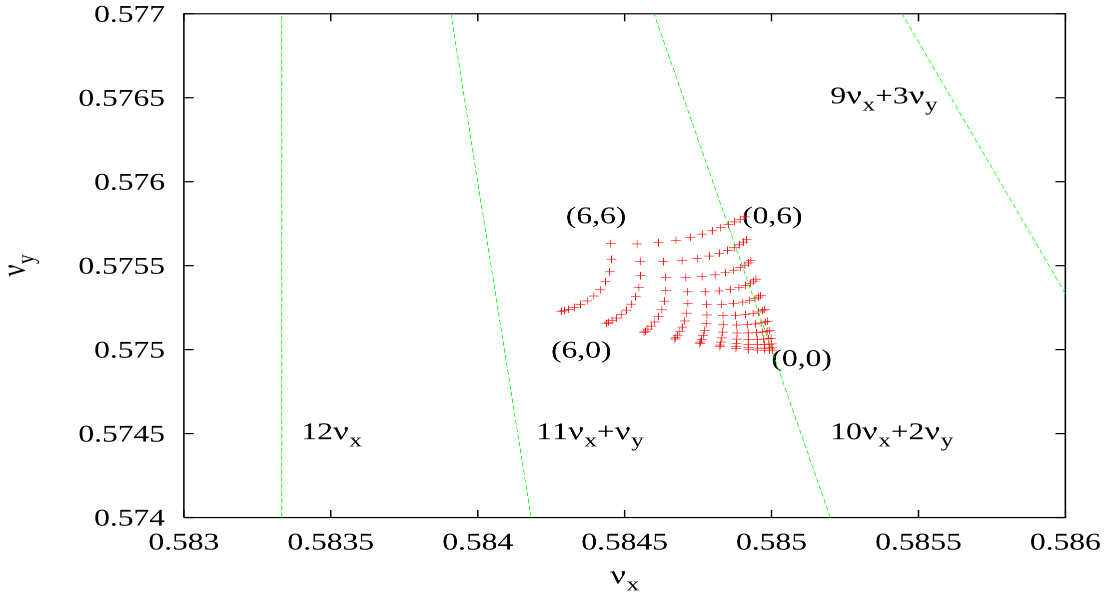
$$F(x,y) = W\left[\frac{x+iy}{\sqrt{2(\sigma_x^2-\sigma_y^2)}}\right] - \exp[-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2}]W\left[\frac{x\frac{\sigma_y}{\sigma_x} + iy\frac{\sigma_x}{\sigma_y}}{\sqrt{2(\sigma_x^2-\sigma_y^2)}}\right] \tag{13}$$

where W is the complex error function. Similar expressions with $x \leftrightarrow y$ result if $\sigma_y > \sigma_x$.

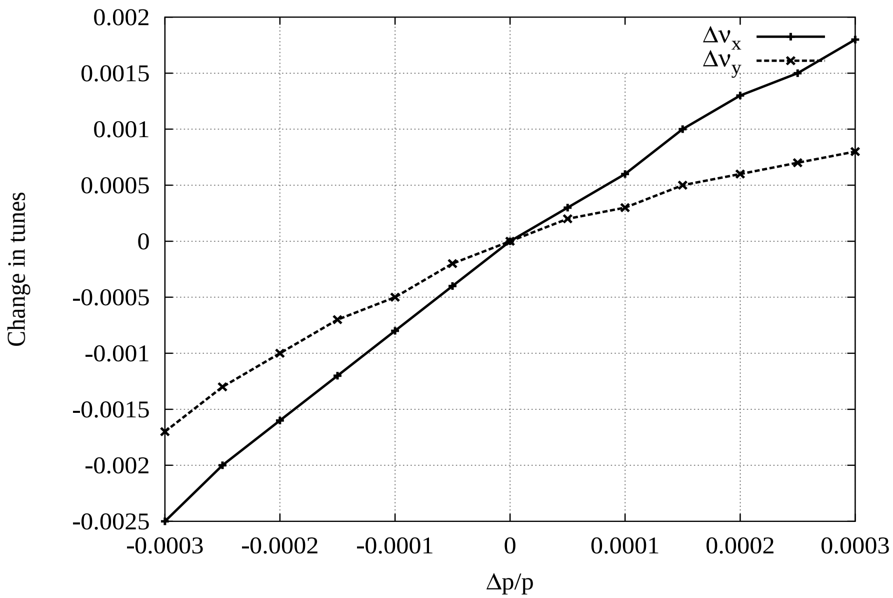
Footprint and Dynamic Aperture(DA) tracking

- PC=1000GeV, working point:(0.585,0.575), Linear Chromaticities (5,5) ε_p=3.3 mm.mrad, and beam size at B0 and D0 are 33nm(10^{-6}m)(1σ)
- $\nu_s = 7.03X10^{-4}$, 1442 turns equivalent to 1 Synchrotron period
- Revolution frequency 47.713KHZ , T=$0.02095X10^{-3}$ sec.
  - 10^5 turns equivalent to 2 second
  - 10^6 turns equivalent to 21 second
  - $1.8X10^8$ turns equivalent to 1 hour
- Tevatron latest lattice, including all the magnet reshuffles in the B0 and D0, the new feeddown circuits for both proton and anti-proton's helixes
- **The tune shift** experienced by zero amplitude particles - one way to parametrize the strength of a kick with separated beams
- **The footprint** is a good measure of the strength of the nonlinearity
- **The Dynamic Aperture** is calculated by launching particles at several angles in $x - y$ space, usually thirteen spaced apart by 7.5°, from 0° to 90°. The radial dynamic aperture at each angle is calculated from the largest stable amplitude below which all amplitudes are stable. From these values at different angles we extract an angle averaged dynamic aperture and a minimum value.
- Trackings were done mostly by MAD, usually it takes one or two days for a job of 100,000 turns' tracking

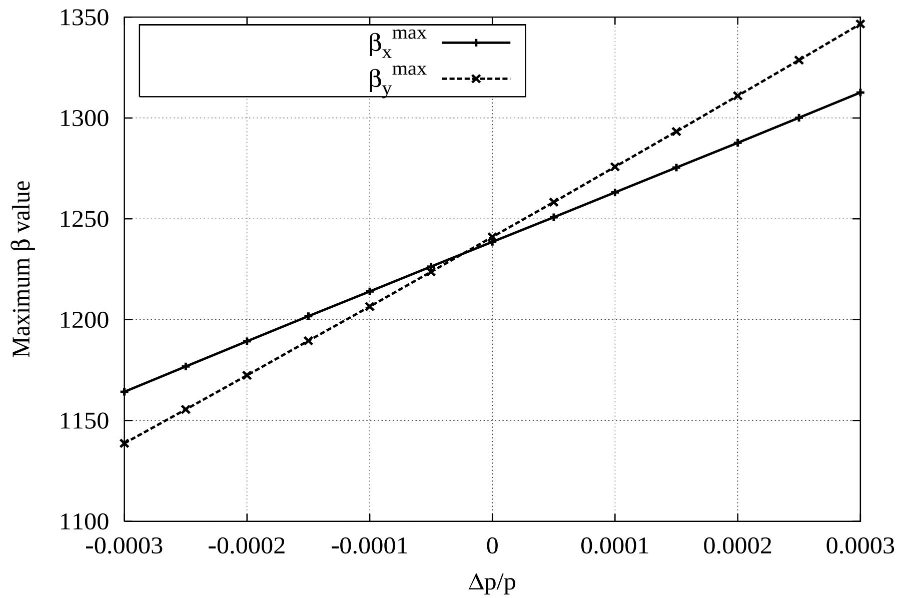
Tune fooprint of a single beam with IR errors and chromaticity sextupoles.



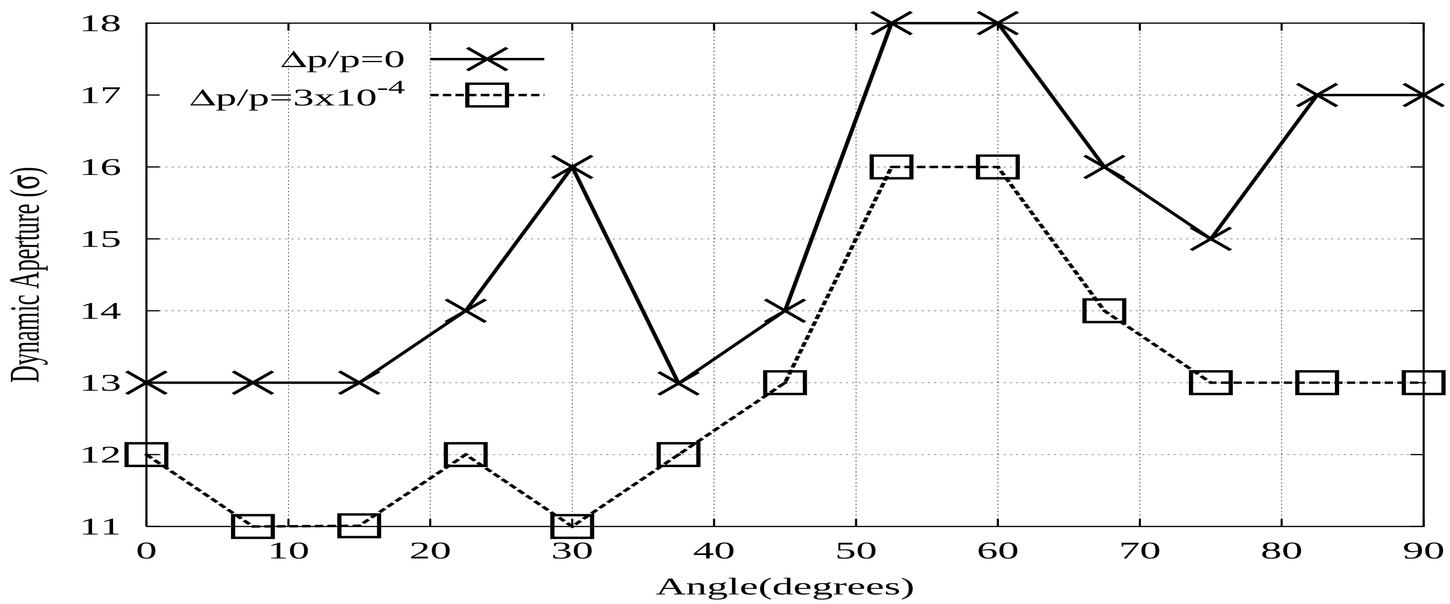
Change in tunes with $\Delta p/p$



Maximum beta values vs $\Delta p/p$



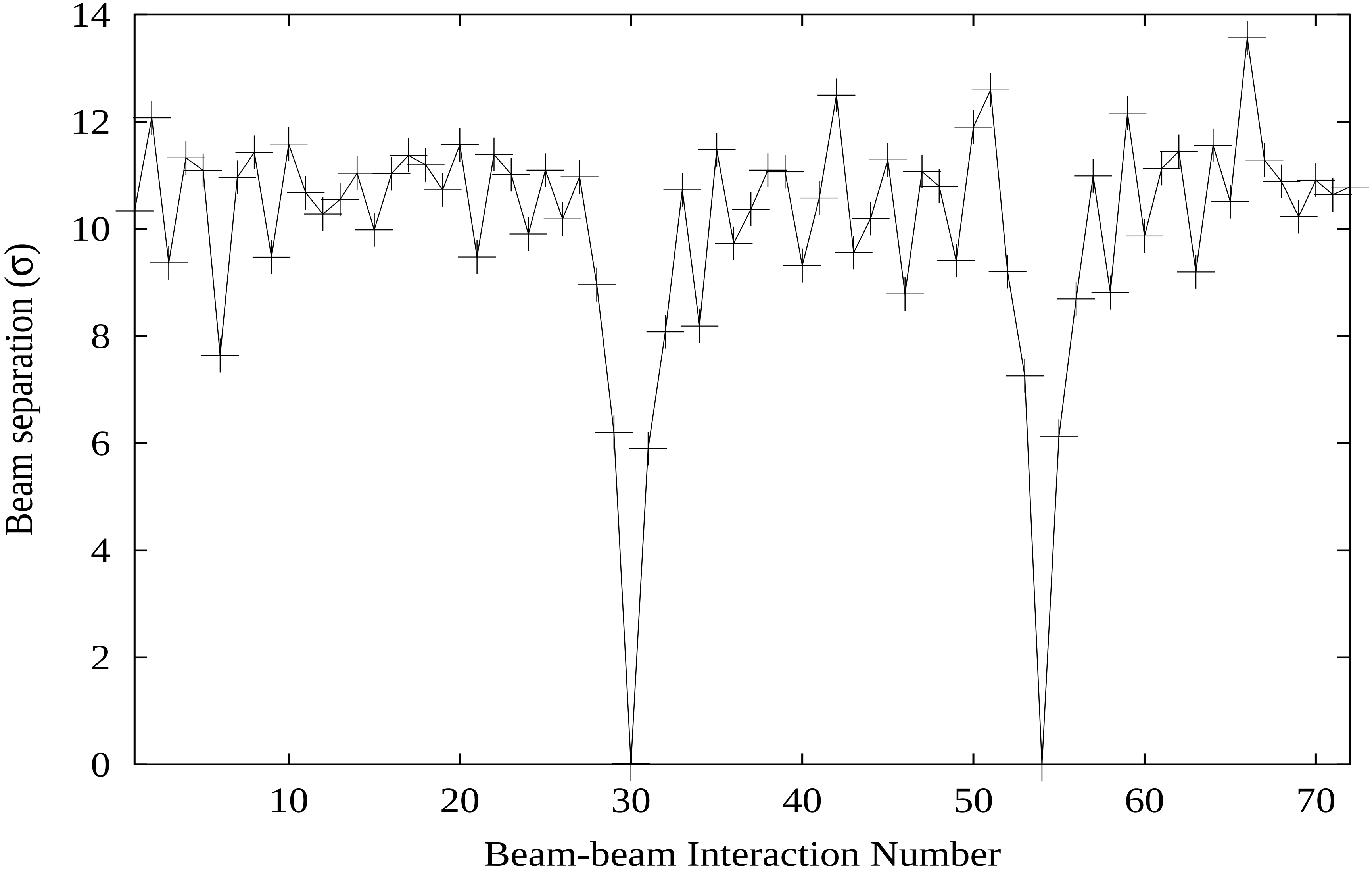
Dynamic aperture of a single beam



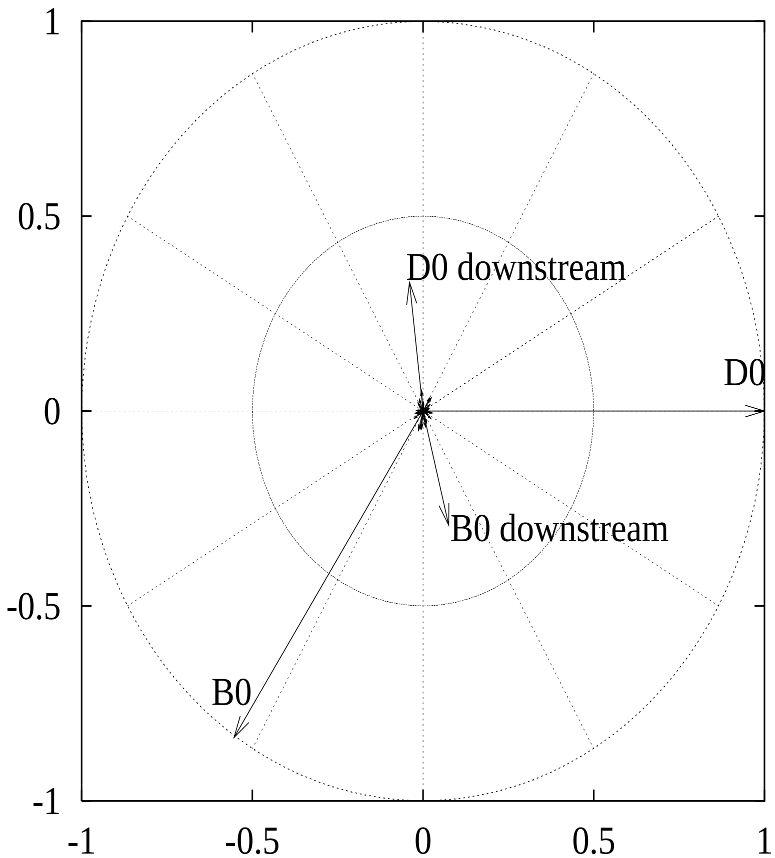
6D dynamic aperture of a single beam at three values of the momentum deviation after 10^5 turns.

	Average DA $\langle DA \rangle$	Minimum DA DA_{min}
dp/p = 0	15.2	13.0
dp/p=1×10^{-4}	14.3	12.0
dp/p=3×10^{-4}	12.9	11.0

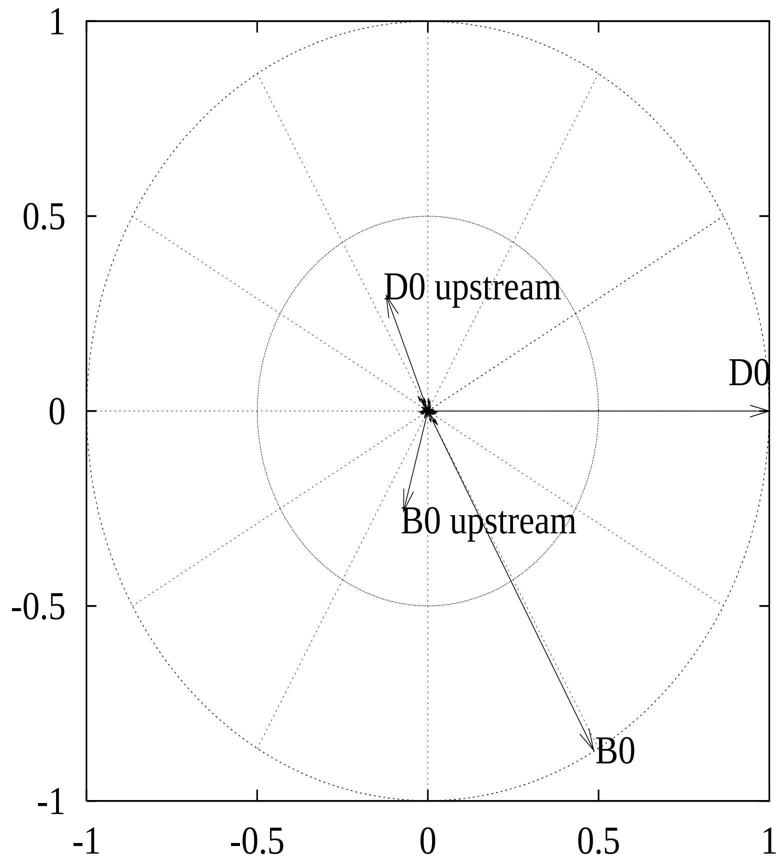
Separation at beam-beam encounters; Bunch 6



Phasor diagram: horizontal kick

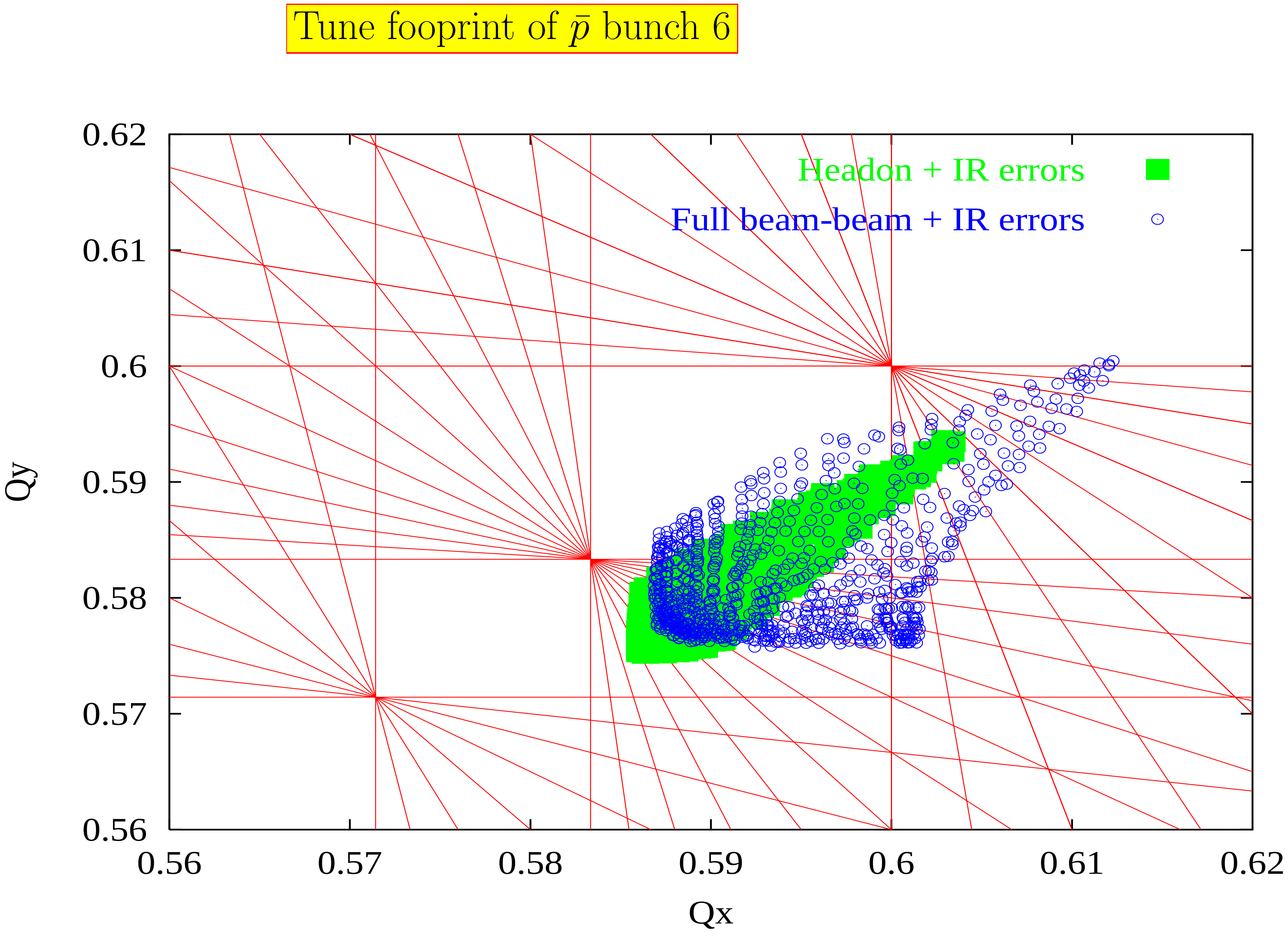


Phasor diagram: vertical kick

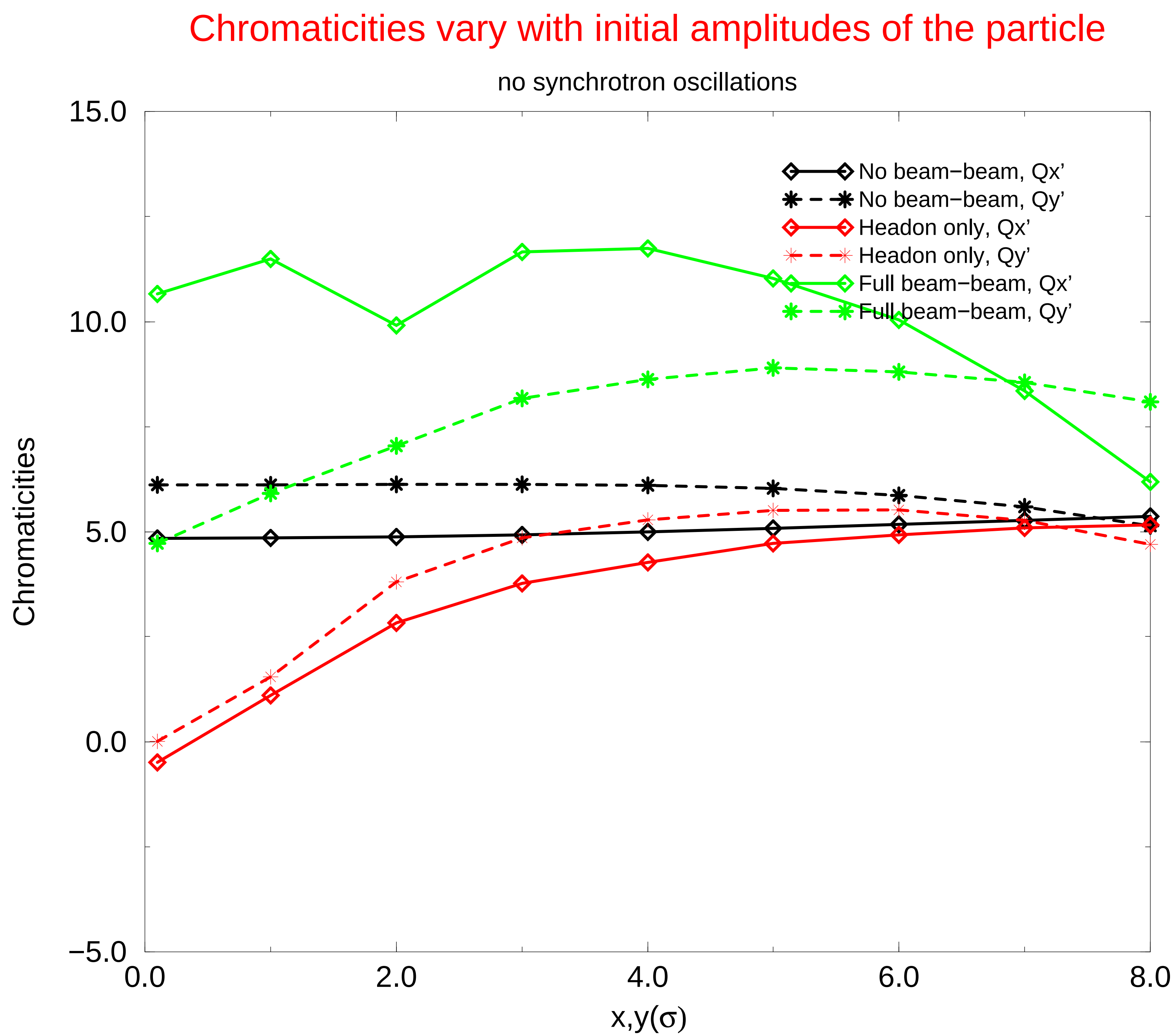


The length of each phasor : the tuneshift due to this kick at zero amplitude

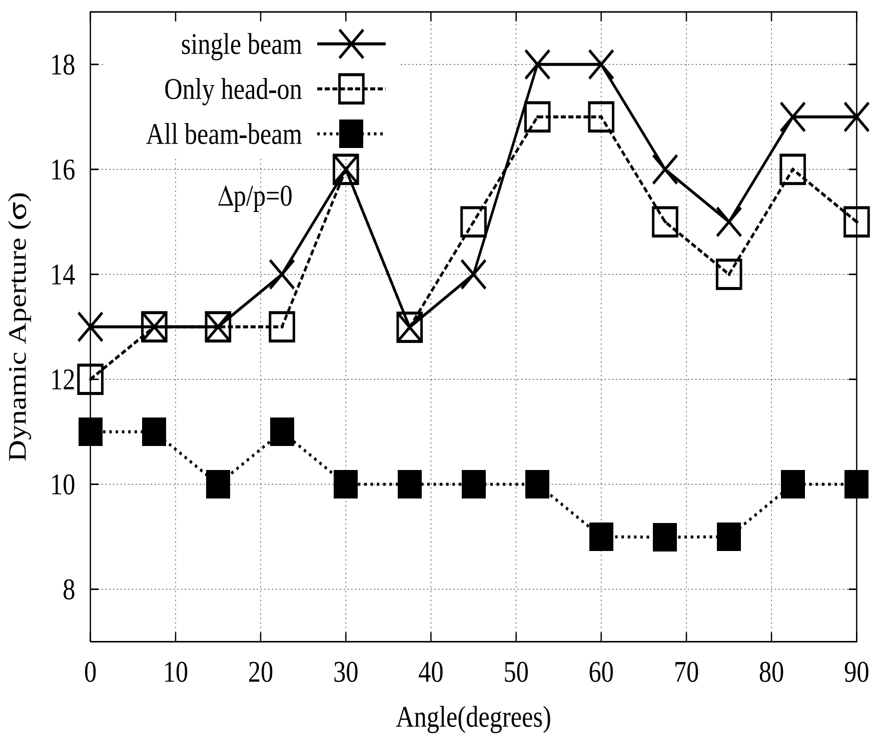
The angle of each phasor: the phase advance,modulo 2π, from D0.



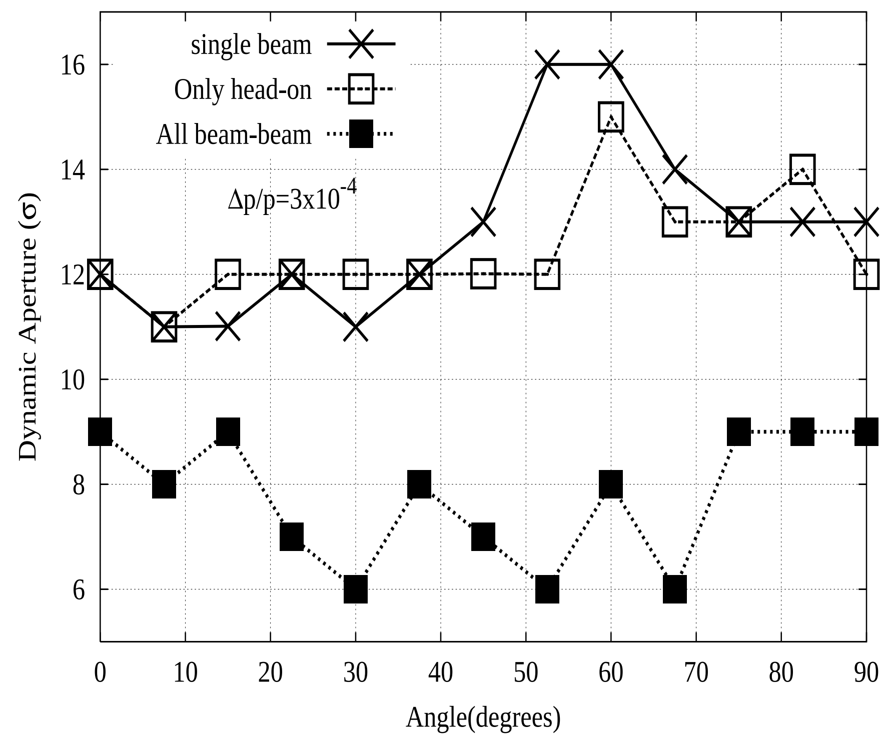
Tune fooprint of $\bar{p}$ bunch 6
Headon + IR errors
Full beam-beam + IR errors
Qy
Qx
0.56
0.57
0.58
0.59
0.6
0.61
0.62


Chromaticities vary with initial amplitudes of the particle
no synchrotron oscillations
15.0
10.0
5.0
0.0
−5.0
0.0
2.0
4.0
6.0
8.0
Chromaticities
x,y(σ)
No beam−beam, Qx'
No beam−beam, Qy'
Headon only, Qx'
Headon only, Qy'
Full beam−beam, Qx'
Full beam−beam, Qy'

4D dynamic aperture of pbar bunch 6



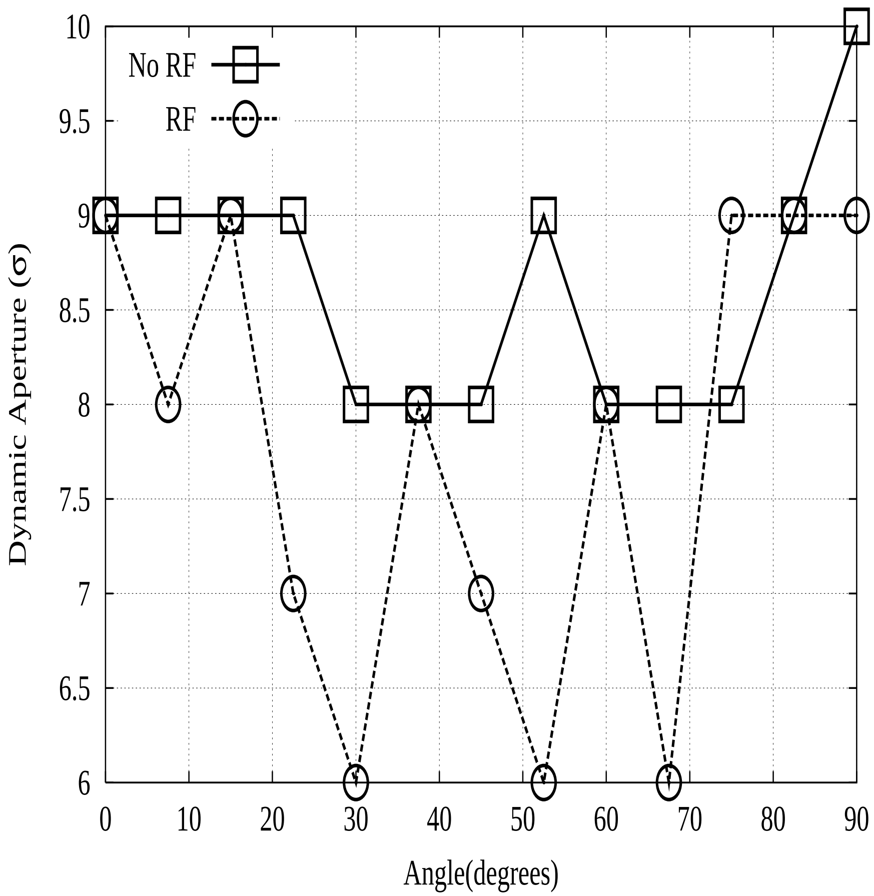
6D dynamic aperture of pbar bunch 6



6D DA with all Beam-beam at different values of $\Delta p/p$ after 10^5 turns and 10^6 turns

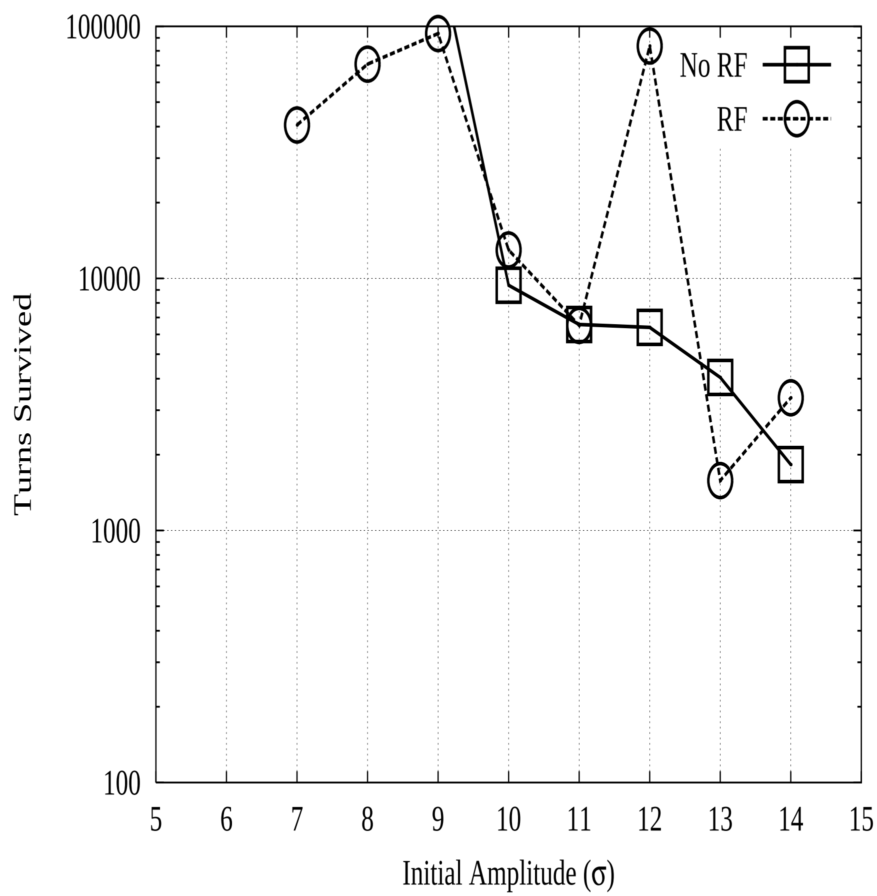
	Average DA $\langle DA \rangle$	Minimum DA DA_{min}
dp/p = 0	10.0	9.0
dp/p=3×10^{-4} (No RF)	8.6	8.0
dp/p=3×10^{-4}	7.7	6.0
	10^6 turns	
dp/p=3 $\times10^{-4}$	5.4	4.0

DA with RF on and off



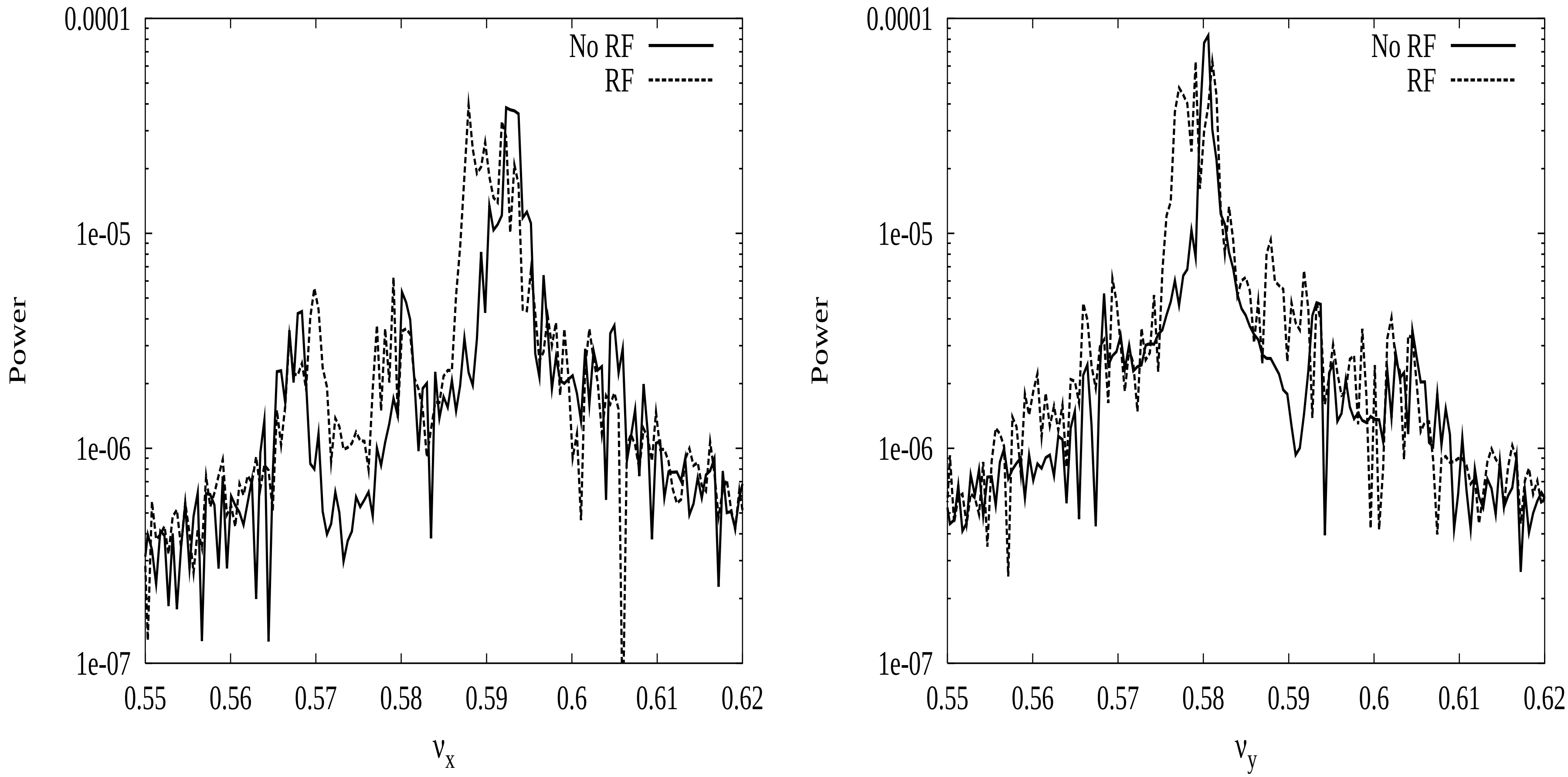
$\Delta p/p = 3 \times 10^{-4}$

Survival time with RF on and off



Particles at an initial angle of 52.5°

Power spectrum of the horizontal and vertical motions



The particle at (9.25σ, 52.5°)

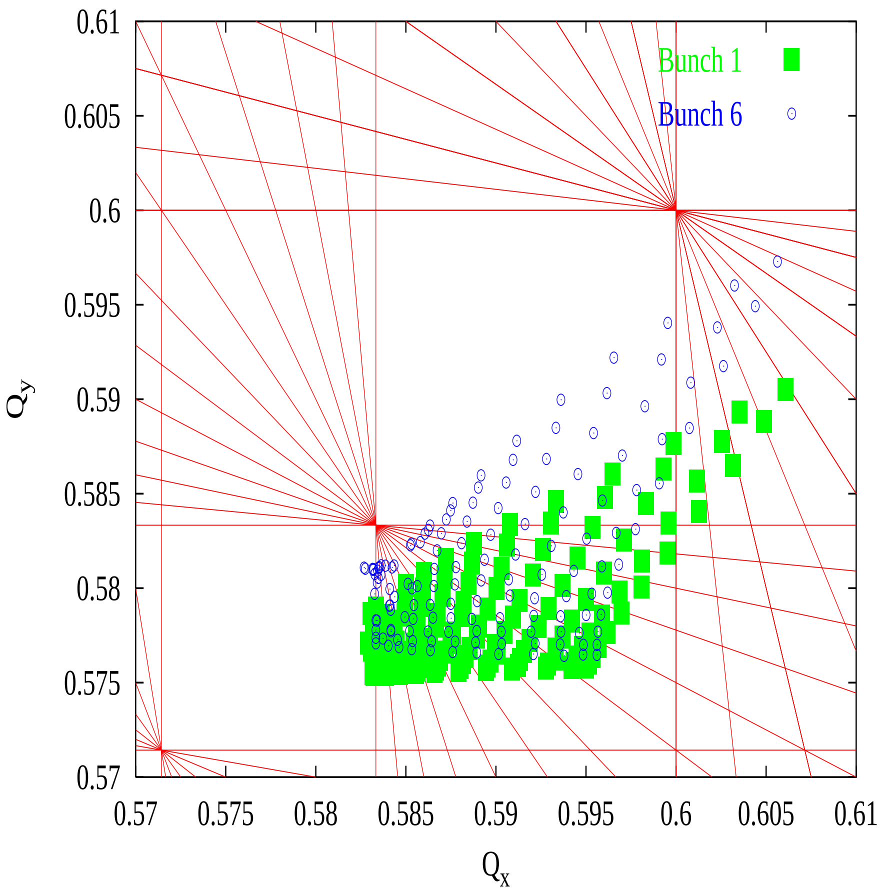
Which experiences the largest change in the action due to synchrotron oscillations

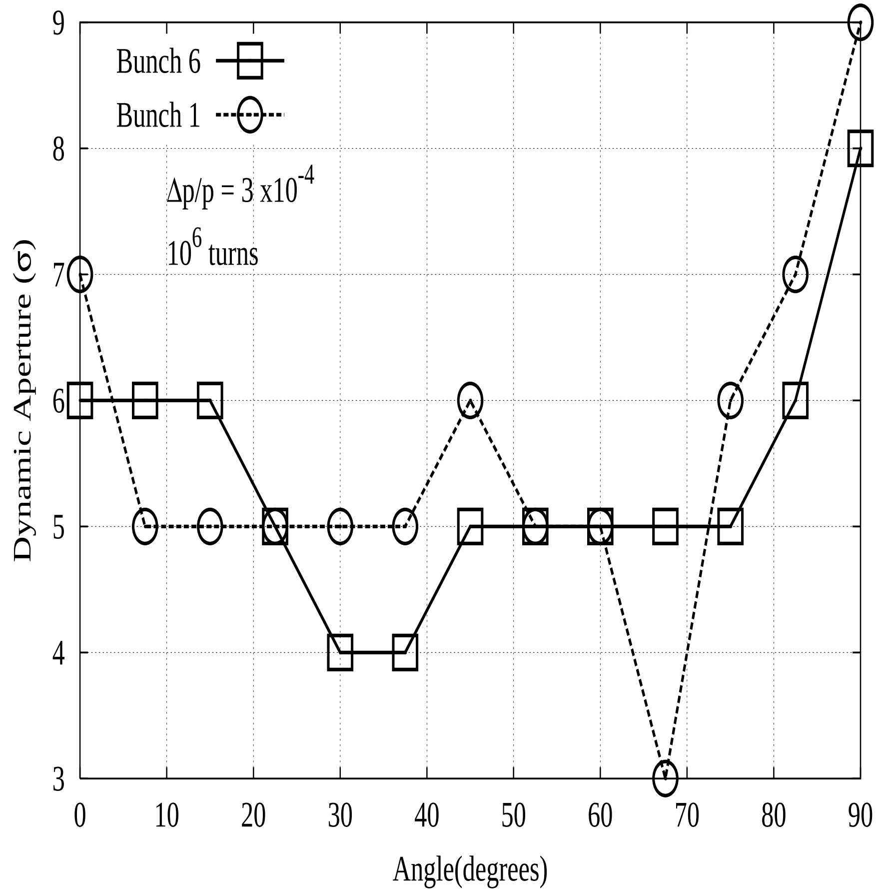
Amplitude (σ)	Angle (degrees)	Resonance Families (m_x, m_y); m_s	$\Delta J_{sum}^{max}/J_{sum}$
9.25	52.5	(1, -1); $m_s = -3, -2, -1$ (7, 5); $m_s = -2, ..., 3$ (8, 4); $m_s = -3$	0.42
10.0	52.5	(1, -1); $m_s = -3, ..., 3$ (3, 9); $m_s = -3, ..., 3$ (4, 8); $m_s = -3, ..., 0$	0.33
9.0	52.5	(1, -1); $m_s = -3, -2$ (8, 4); $m_s = -3, ...3$	0.18
9.25	22.5	(1, -1); $m_s = -3$ (8, 4); $m_s = -2, ...3$	0.14
9.0	22.5	(1, -1); $m_s = -3, -2$ (8, 4); $m_s = -3, ...3$	0.11
9.5	22.5	(1, -1); $m_s = -3$ (8, 4); $m_s = -2, ...2$	0.08
10.0	22.5	(1, -1); $m_s = -3, -2$ (7, 5); $m_s = 1, 2, 3$ (8, 4); $m_s = -3, -2, -1$	0.06
9.75	22.5	(1, -1); $m_s = -3$ (8, 4); $m_s = -3, ...2$	0.04
9.5	52.5	(1, -1); $m_s = -3$ (8, 4); $m_s = -3, ..., 3$	-0.02
9.75	52.5	(1, -1); $m_s = -3, -2, -1$ (7, 5); $m_s = -3, ..., 3$	-0.02

Table 2: Synchro-betatron resonance families associated with particles near the dynamic aperture. The resonance families are arranged in descending order of the relative change in action $\Delta J_{sum}^{max}/J_{sum}$.

Tune footprint for bunches 1 and 6

Dynamic aperture of bunch 1 and 6



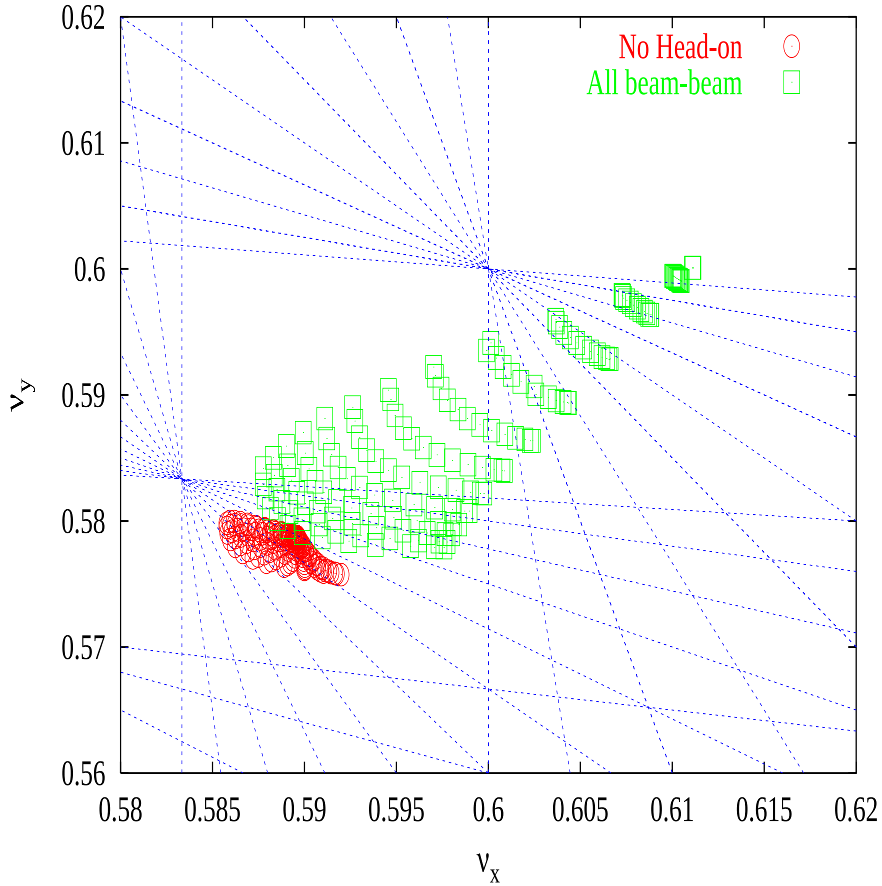


All particles had $\Delta p/p = 3 \times 10^{-4}$. Tracking for 10^6 turns

	Bare tunes	4D DA ($\langle DA \rangle$, DA_{min})	6D DA ($\delta_p = 3 \times 10^{-4}$) ($\langle DA \rangle$, DA_{min})
A0	0.585, 0.575	(10.0, 9.0)	(7.8, 6.0)
A1	0.575, 0.569	(9.2, 7.0)	(5.1, 4.0)
A2	0.577, 0.571	(9.3, 8.0)	(7.5, 6.0)
A3	0.579, 0.573	(9.4, 9.0)	(8.1, 7.0)
A4	0.583, 0.577	(9.8, 9.0)	(7.6, 6.0)
A5	0.585, 0.579	(9.6, 8.0)	(7.5, 7.0)
A6	0.587, 0.581	(9.5, 8.0)	(7.5, 6.0)
A7	0.575, 0.585	(11.0, 9.0)	(8.6, 7.0)
A8	0.577, 0.587	(10.7, 9.0)	(8.4, 8.0)
A9	0.579, 0.589	(10.5, 9.0)	(7.6, 5.0)
A10	0.581, 0.591	(10.0, 8.0)	(7.0, 5.0)
A11	0.583, 0.593	(9.5, 6.0)	(4.8, 3.0)
A12	0.585, 0.595	(8.5, 6.0)	(1.9, 1.0)
A13	0.551, 0.561	(10.9, 9.0)	(7.2, 5.0)
A14	0.553, 0.562	(10.7, 9.0)	(6.2, 5.0)
A15	0.555, 0.564	(10.2, 9.0)	(7.2, 6.0)
A16	0.556, 0.566	(9.9, 8.0)	(5.7, 3.0)
A17	0.558, 0.568	(11.0, 9.0)	(5.4, 3.0)
A18	0.560, 0.570	(10.5, 8.0)	(5.4, 3.0)

Table 3: Dynamic aperture, both 4D and 6D, calculated after 10^5 turns at different tunes. All beam-beam interactions were included. A0 is the nominal tune, A1, A2, A17 and A18 are close to 7th order resonances while A12 is close to 5th order resonances. We observe that at tunes away from these low order resonances the dynamic aperture does not change significantly.

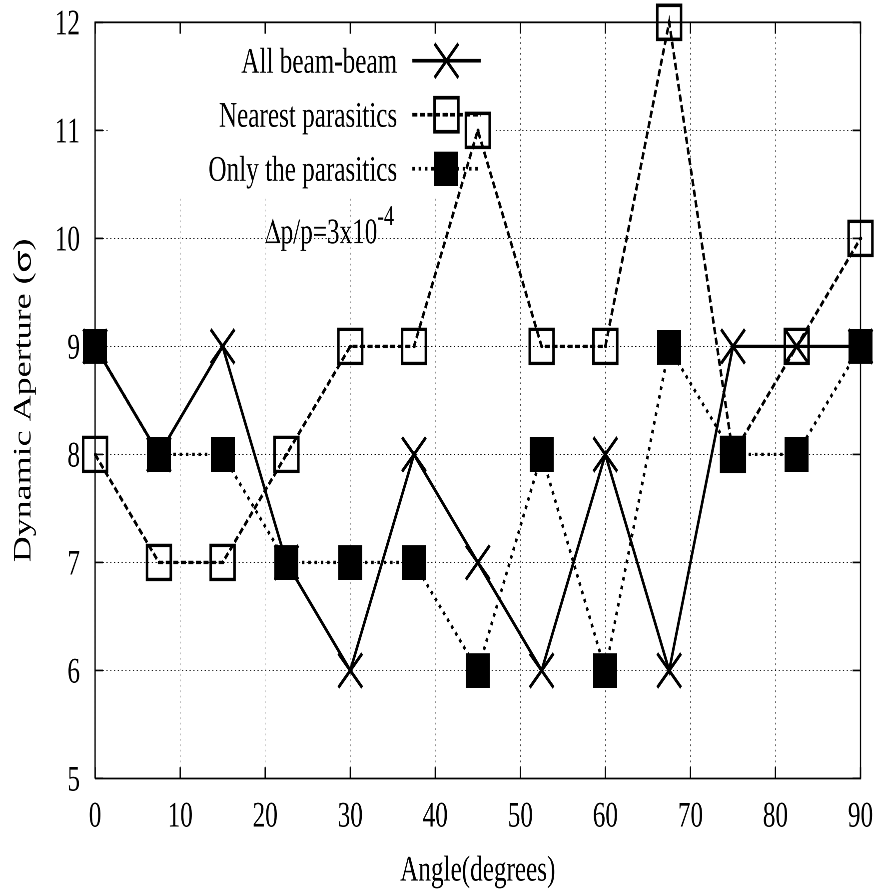
Footprint for bunches 6



with (i) all parasitic interactions but no head-on

with (ii) all the beam-beam interactions

Dynamic aperture of bunch 6



(1) all beam-beam interactions,

(2) head-on and nearest parasitics

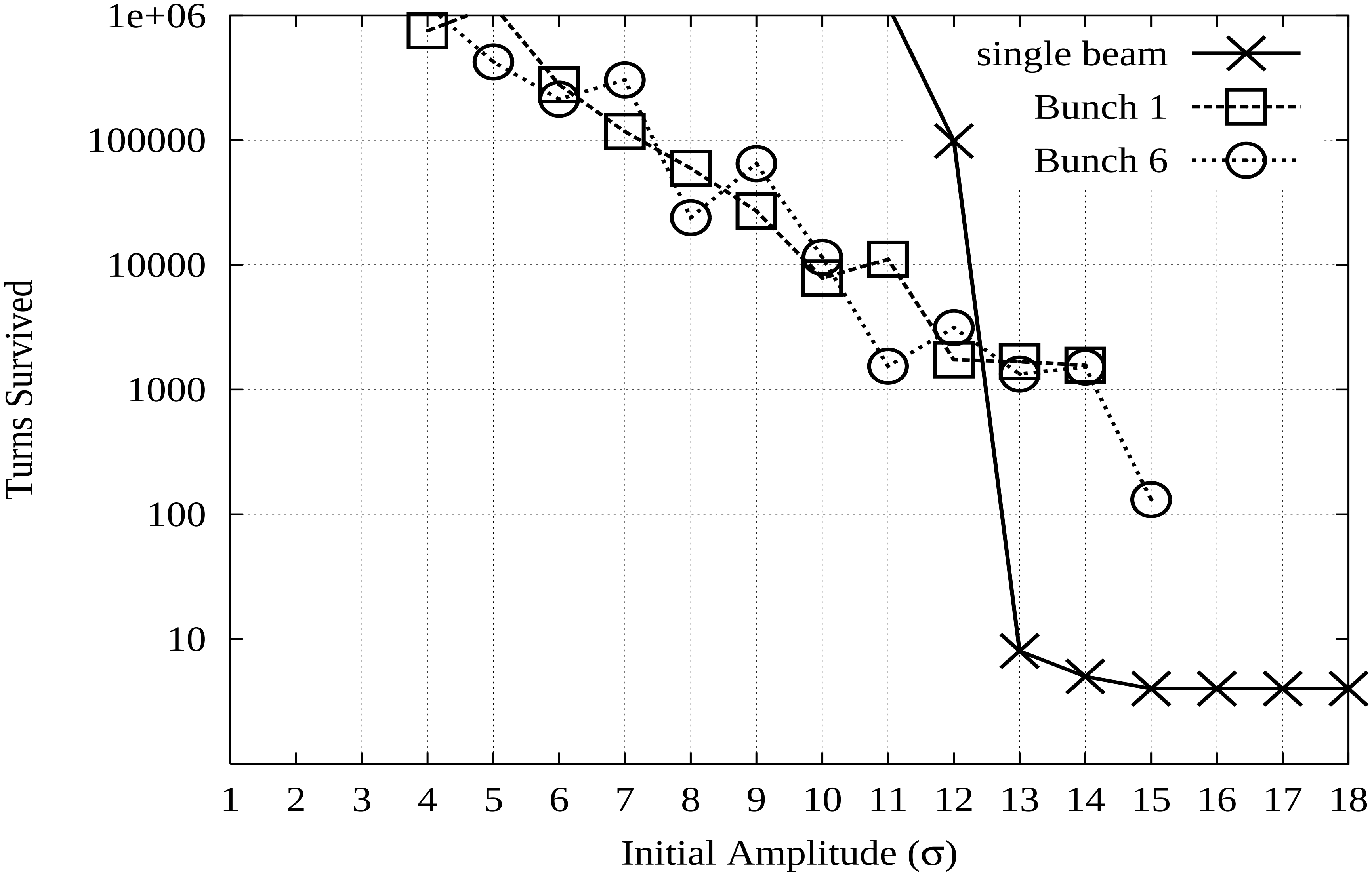
(3) only the parasitics.

Summary of different cases

Bunch 6: $\nu_x = 0.585$, $\nu_y = 0.575$ DA after 10^5 turns		
	$(\langle DA \rangle, DA_{min})$ [4D] $\Delta p/p = 0$	$(\langle DA \rangle, DA_{min})$ [6D] $\Delta p/p = 3 \times 10^{-4}$
IR errors	(15.2, 13.0)	(12.9, 11.0)
Head-on and IR errors	(14.5, 12.0)	(12.5, 11.0)
Head-on, nearest PCs, IR errors	(10.5, 9.0)	(8.9, 7.0)
Head-on, nearest PCs at 10σ, IR errors	(13.5, 12.0)	(10.2, 8.0)
Only the parasitics, IR errors	(10.2, 9.0)	(7.7, 6.0)
All beam-beam, IR errors	(10.0, 9.0)	(7.7, 6.0)
$(\langle DA \rangle, DA_{min})$ for bunches 6, 1 and 12 [6D, $\Delta p/p = 3 \times 10^{-4}$]		
	10^5 turns	10^6 turns
Single beam	(12.9, 11.0)	(12.3, 11.0)
Bunch 6: all beam-beam	(7.7, 6.0)	(5.4, 4.0)
Bunch 1: all beam-beam	(7.8, 6.0)	(5.6, 3.0)
Bunch 12: all beam-beam	()	

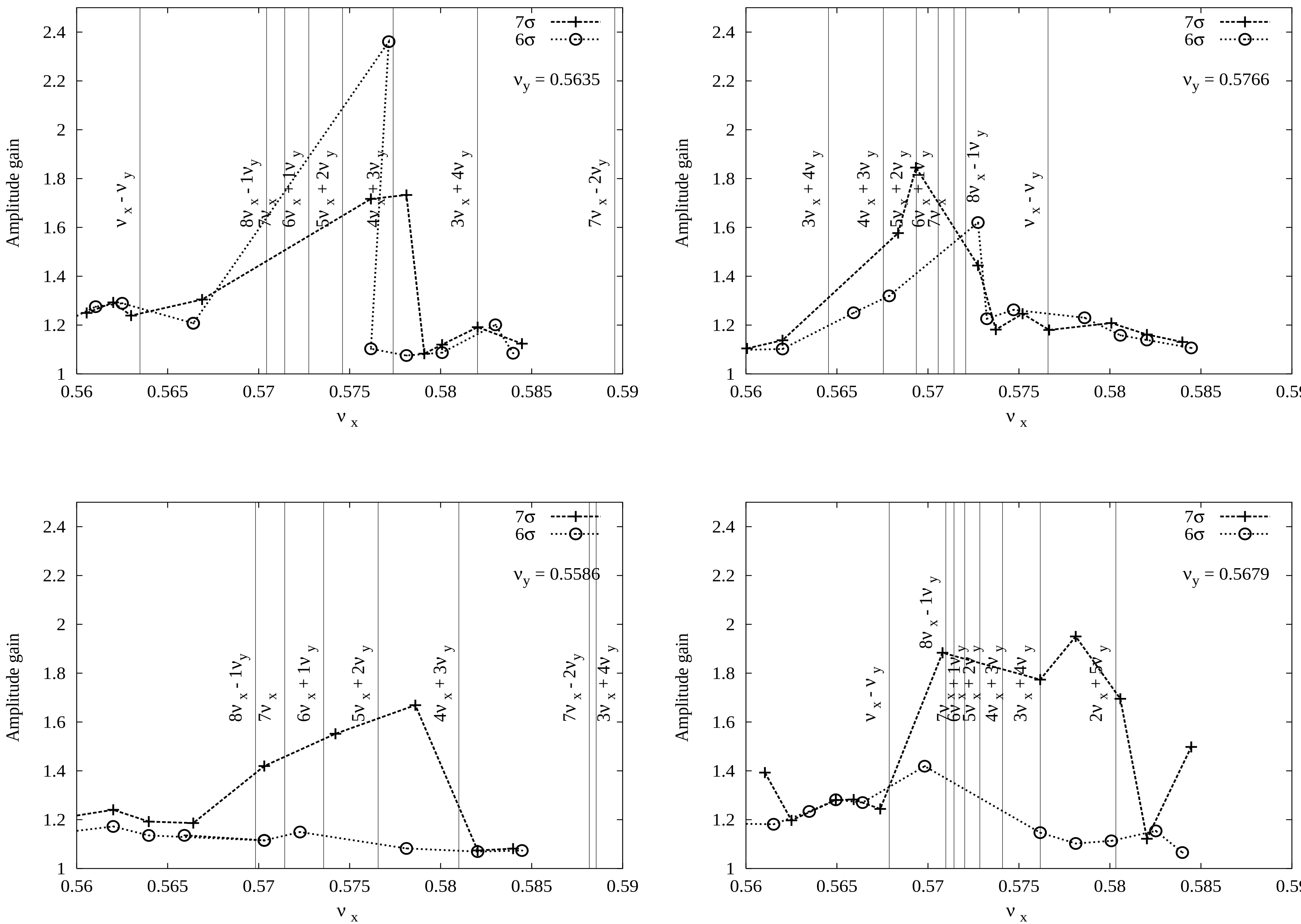
Table 4: The average and minimum 6D dynamic aperture with various configurations of beam-beam interactions. Note that the dynamic aperture with only the parasitics is nearly the same as that with all the beam-beam interations. The head-on interactions therefore are dominated by the parasitics. Also shown are the average and minimum 6D dynamic aperture for bunches 1 and 12 at the edges of the bunch train compared with bunch 6 in the middle of the train.

Survival plots for three cases along the most unstable angle in each case

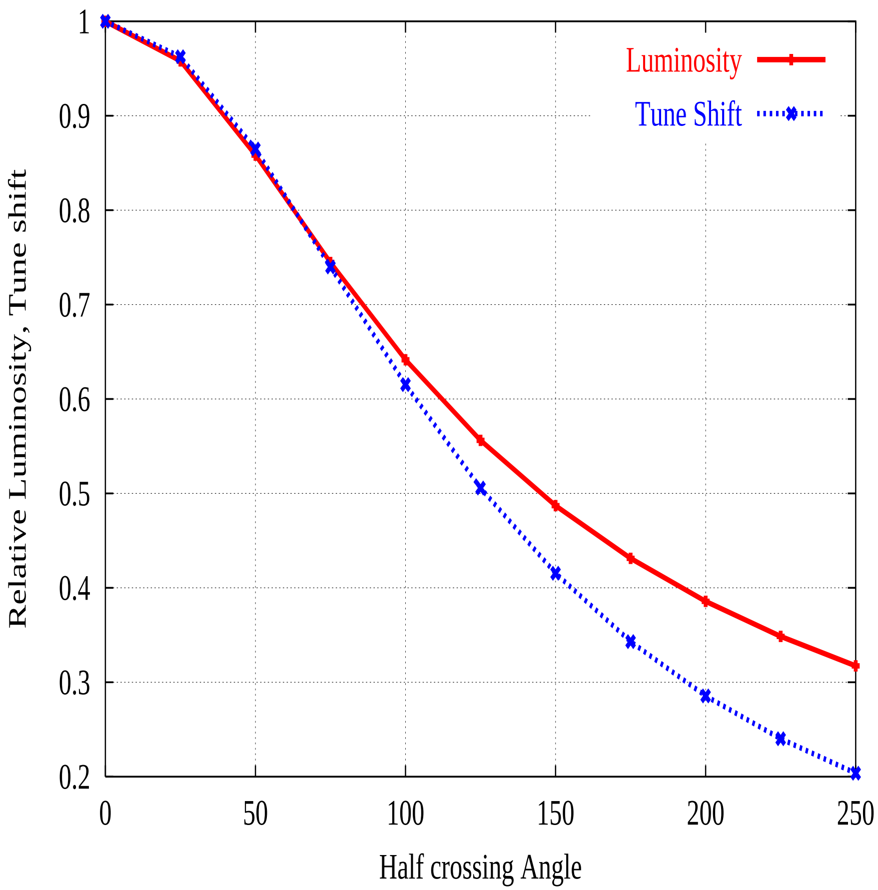


Particles with a momentum deviation of $\Delta p/p = 3 \times 10^{-4}$

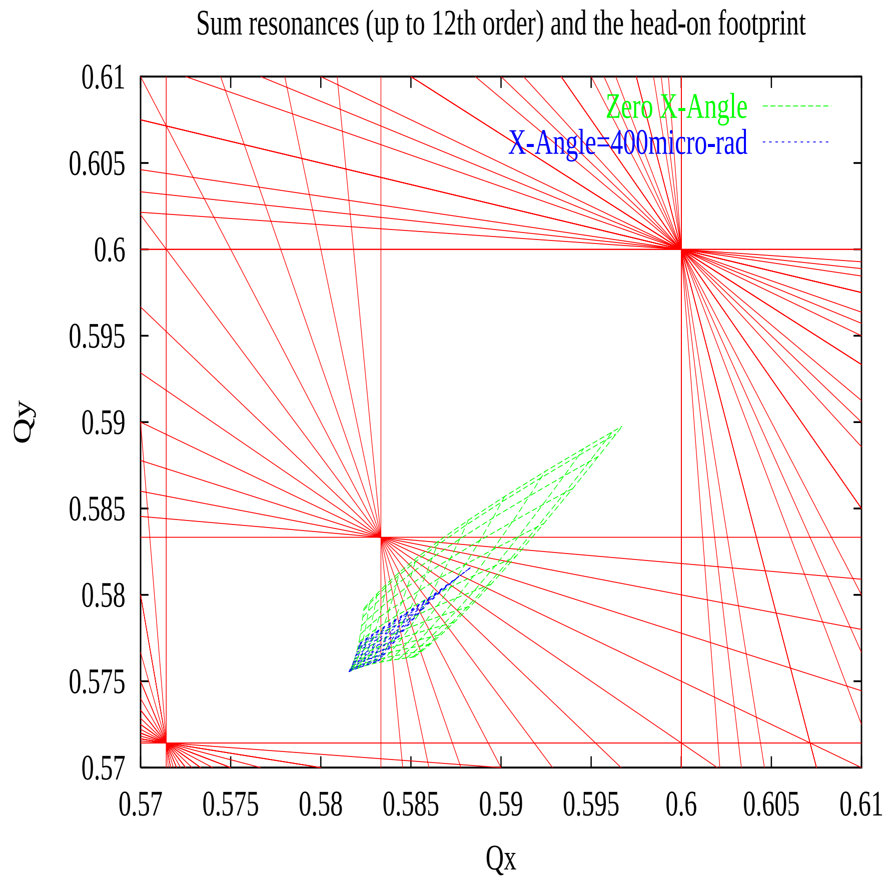
Tune scans around resonances and the amplitude growth around these resonances



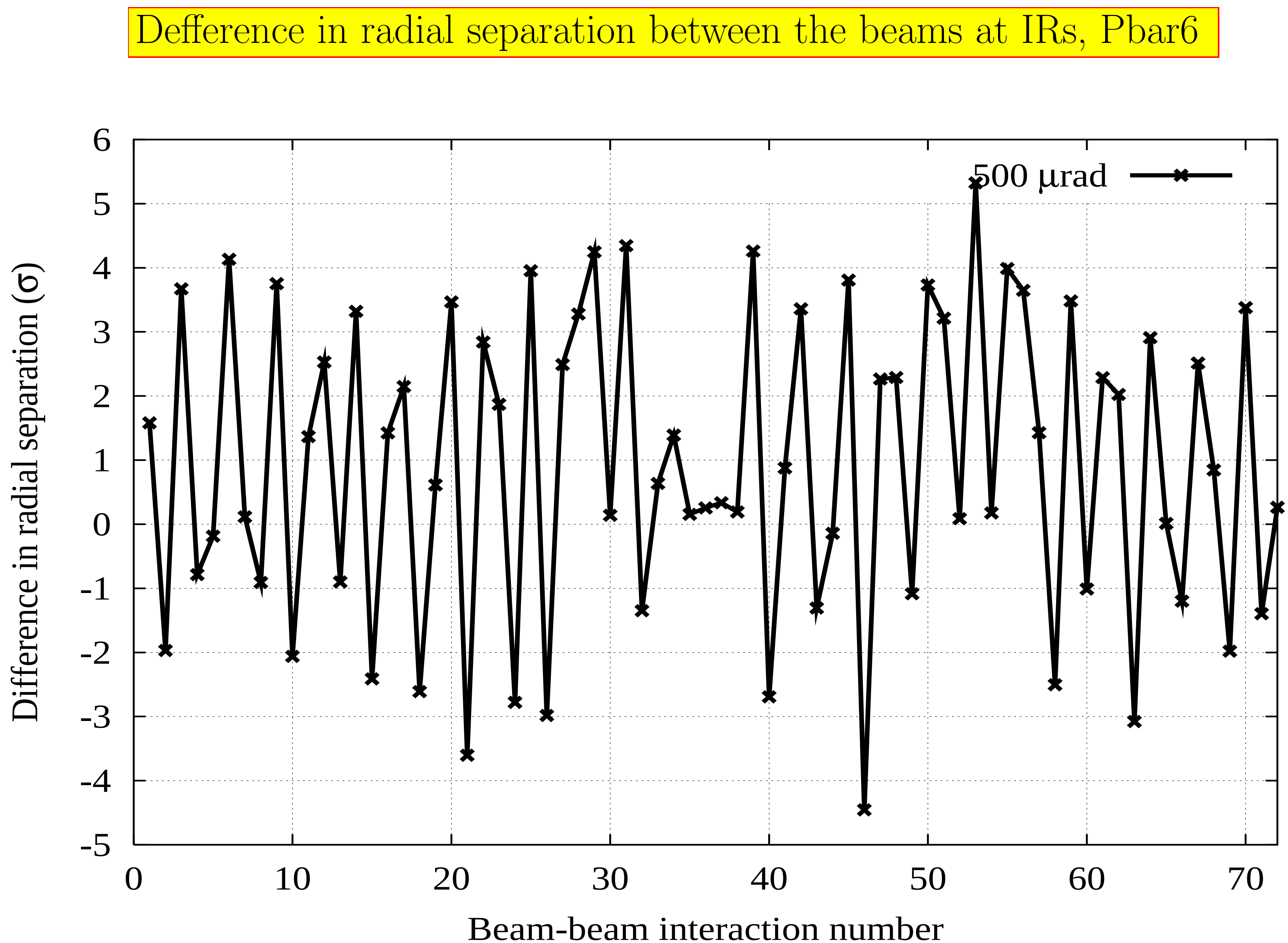
The impact of a crossing angle



The beam-beam tune footprint

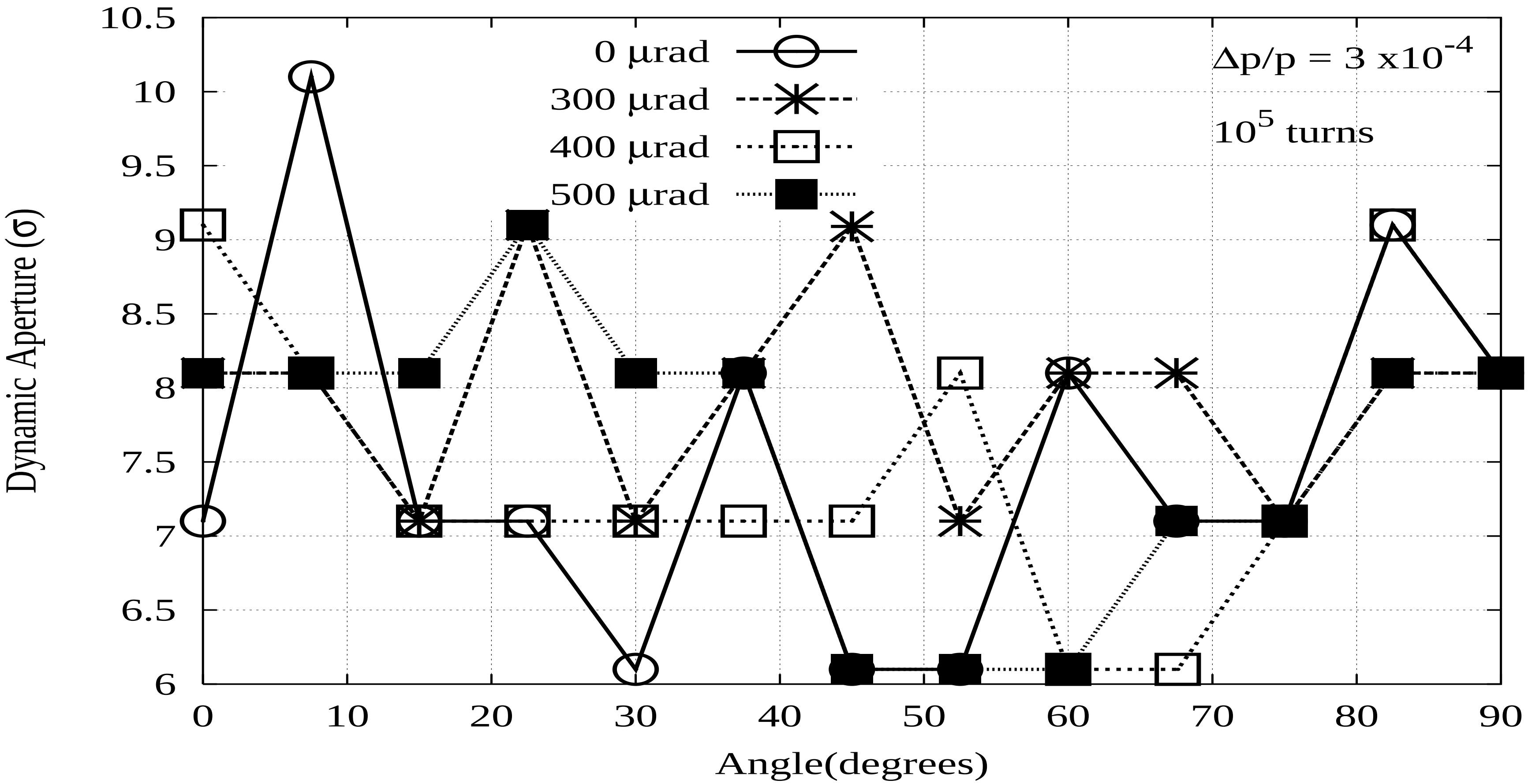


All values are normalized to the value at zero crossing angle



Defference in radial separation between the beams at IRs, Pbar6
500 μrad
Difference in radial separation (σ)
Beam-beam interaction number
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
0
10
20
30
40
50
60
70

DA with crossing angles of RUN IIa, Full Beam-beam+IR errors



6D DA after 10^5 turns at three different crossing angles

Crossing angle (μ rad)	Average DA $\langle DA \rangle$	Minimum DA DA_{min}
0	7.5	6.0
300	7.9	7.0
400	7.5	6.0
500	7.6	6.0

Conclusions

- *What is the dynamic aperture (DA) with all the beam-beam interactions ?*

  At design parameters and after 10^6 turns,**the DA for bunch 6 is about 8σ (4D) and about 5σ (6D, $\Delta p/p = 3 \times 10^{-4}$, $\nu'_x = \nu'_y = 5$).** The 6D value is smaller than the aperture limitation set by the primary collimators. The dynamic aperture does not appear to reach an asymptotic value even after 10^6 turns but instead keeps decreasing with the number of turns followed. This contrasts with the dynamic aperture for single beams which does not change much after 10^5 turns.

- *Is the DA sensitive to the choice of tunes?*

  A tune scan around the nominal working point (the tune values can be seen in Figure **??**) showed that **the DA does not change significantly** as long as the tunes are sufficiently far from the 5th and 7th order resonances. Reversing the tunes ($\nu_x = 0.575$, $\nu_y = 0.585$) results in a marginally larger value of the DA by 1σ.

- *How important are the head-on interactions compared to the long-range interactions?*

  The tune footprint is largely determined by the head-on interactions. However **they have very little influence on the dynamic aperture**. Thus the dynamic aperture with only the parasitic interactions is nearly the same as that with the parasitic and the head-on interactions. This is observed to be true at several other tunes in the vicinity of the working point. **Hence it is not obvious that compressing the footprint would improve the dynamic aperture.**

- *Is there a dominant group of long-range interactions ?*

  Yes, of the seventy long-range interactions **the four interactions nearest to the two IPs are the dominant group**. The 4D DA with only these long-range interactions is nearly the same as with all the interactions while the 6D DA is about 1σ larger. This also indicates that the synchro-betatron resonances driven by the other long-range interactions are important.

- *Is the DA sensitive to the separation at the nearest parasitics?*

  The average 4D DA increases by about 1.5σ when the separation at these locations is increased from 6σ to 10σ. The increase in the 6D DA is smaller, only about 0.5σ. This suggests that **the effects of the synchro-betatron resonances driven by the long-range do not decay very rapidly with the beam separation.** This in turn may be because the amplitude dependent chromaticity due to the long-range interactions is relatively insensitive to the beam separation at large amplitudes. Reducing the longitudinal emittance would be helpful in increasing the 6D DA of a bunch.

This result in 6D is somewhat surprising. This begs the question then why the beam-beam interactions in Run I, where the separations were of the order of 10σ, did not usually pose a serious limitation? The answer may lie in the details; higher proton bunch intensities in this study, differences in Twiss functions at the parasitic interactions, particularly the dispersion, and phase advances between the interactions.

- *What are the mechanisms for amplitude growth ?*

  We have found with 4D tracking that there is no evidence of wide-spread diffusion close to the stable boundary. Instead, the motion near the boundary is characterized by thin chaotic layers. At this boundary the survival time depends very sensitively on the initial coordinates. Thus in a pair of particles separated by 0.5μm (0.014σ), one might survive for more than 10^5 turns while the other is lost within 10^4 turns.

  A signature of eventual particle loss appears to be tune changes > 0.001 within 10^4 turns. These changes in tune start long before the particle is lost and are apparently due to the overlapping of twelfth and higher order resonances. Our simulations lend support to the picture of narrow streaming channels in phase space which, relatively slowly, transport particles out to larger amplitudes where other resonances cause fast loss.

- *Which resonances cause fast loss?*

  With a tune scan and 4D tracking we find that at amplitudes of 6σ and 7σ **the seventh order sum resonances, particularly the $4\nu_x + 3\nu_y$ and $5\nu_x + 2\nu_y$ resonances, cause large amplitude growth within 10^3 turns**.

  The synchro-betatron resonances at the nominal working point do not appear to be responsible for fast loss but the additional streaming channels created by the sideband resonances $m_x\nu_x + m_y\nu_y + m_s\nu_s = p$ of the twelfth order sum resonances, in particular $(m_x, m_y) = (3, 9)$, $(4, 8)$, $(7, 5)$, $(8, 4)$ and $|m_s| \leq 3$, destabilize particles that would be stable without synchrotron oscillations over longer time scales.

- *What is the bunch to bunch variation in DA?*

  We have studied bunch 6 (near the center of the bunch train) in some detail and bunch 1 at the head of the train to some extent. **The differences in DA between these two bunches are not significant but bunch 1 may have a marginally smaller DA.**

- *Do compensation schemes work?*

  We have not directly addressed this question in this report. However we do find that **with beam-beam interactions there is no direct correlation between the size of a tune footprint and the dynamic aperture**.

  This suggests that it may be more useful to use the proposed Tevatron electron lens [**?**] to compensate for an "averaged" long-range force from all the parasitic collisions rather than attempting to reduce the footprint in the second stage of this project.

This speculation can be checked with a tracking model that includes the field of the electron lens. A more conventional idea would be to compensate the 7th order resonances which appear to be strongly driven.

- *What effects are not included in this study?*

  We list the important ones: error fields in the arc magnets, misalignments, time dependent effects such as power supply ripple which can modulate the tunes and beam separation, bunch to bunch differences in emittance and intensity.

- *How accurate are the models of the beam-beam interactions?*

  We believe that with the inclusion of bunch length effects, the transverse kicks due to the beam-beam interactions at B0 and D0 are modelled fairly accurately. The MAD model of the beam-beam interaction does not include the energy kicks so these are missing from our tracking. Simulations using other codes for different accelerators, see e.g. Reference [**?**], show that the energy kicks do not have a significant impact.

  The long-range kicks are modelled as point interactions and of course without energy kicks. The justification for ignoring bunch length effects in these interactions is that the beta functions at these locations are not very small so hour-glass effects and the phase advance over the length of the opposing strong bunch should be negligible. That said, it should be noted that point interactions over-emphasize the strength of the kick so the tracking model may over-estimate somewhat the effects due to the long-range interactions.

- *Impact of Crossing angles with 36$\times$ 36 bunches*

  We find that bunch length effects reduce the impact of synchro-betatron resonances when crossing angles are introduced. Once these bunch length effects are included in the interactions at B0 and D0, **the synchro-betatron resonances generated by the crossing angles do not have a significant impact up to crossing angles of 500 μrads**. The dynamics is dominated by the long-range interactions.