Wire Compensation - I

Kick from a round proton beam

$$\Delta x' = -\frac{2N_p r_p}{\gamma_p}\frac{\cos\theta_{PA}}{r_{PA}}\left\{1 - \exp[-\frac{r_{PA}^2}{2\sigma^2}]\right\}$$
$$\Delta y' = -\frac{2N_p r_p}{\gamma_p}\frac{\sin\theta_{PA}}{r_{PA}}\left\{1 - \exp[-\frac{r_{PA}^2}{2\sigma^2}]\right\} \quad (1)$$

The kicks from the wire are

$$\Delta x' = \frac{\mu_0}{2\pi}\frac{I_W L}{(B\rho)}\frac{\cos\theta_{WA}}{r_{WA}}, \qquad \Delta y' = \frac{\mu_0}{2\pi}\frac{I_W L}{(B\rho)}\frac{\sin\theta_{WA}}{r_{WA}} \quad (2)$$

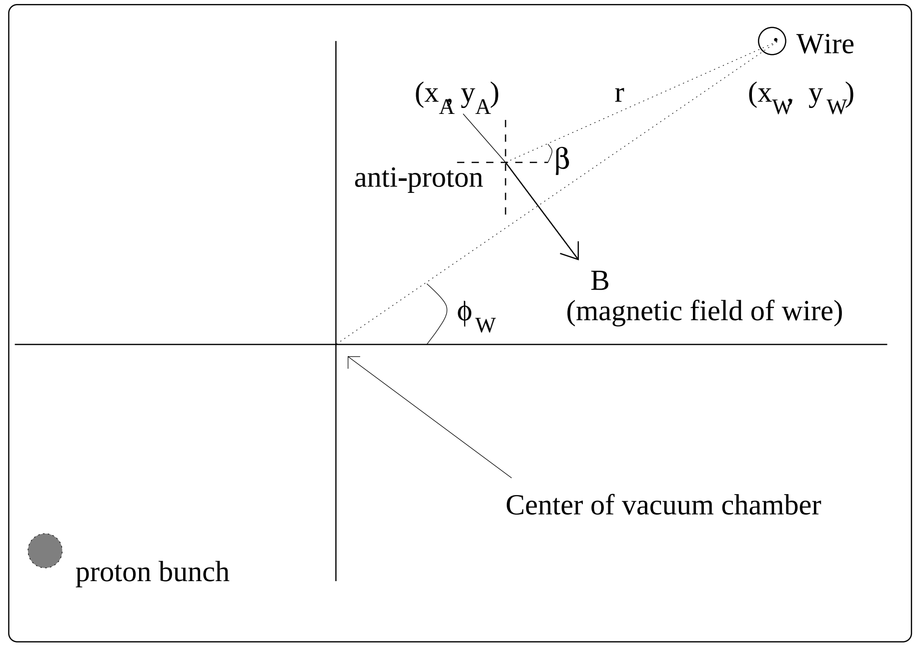
At separations larger than 3σ, the fields cancel provided the current is chosen appropriately.

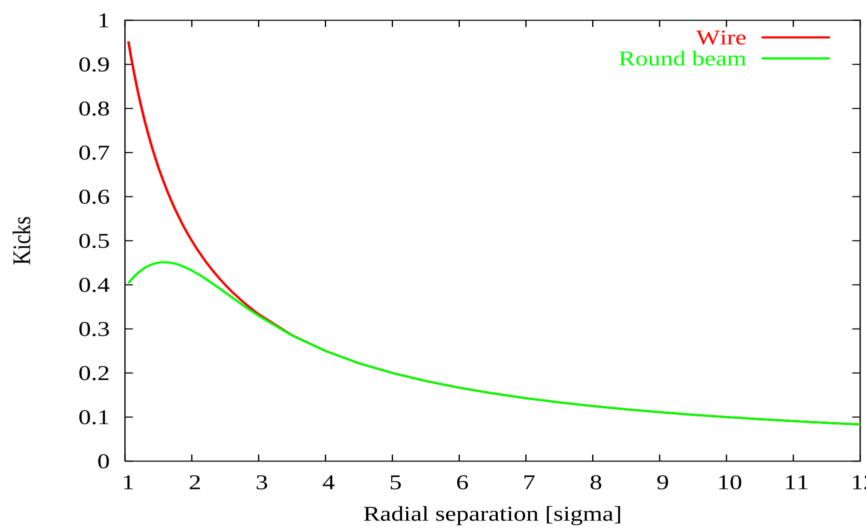
Caveats:

- Exact cancellation requires

$$\theta_{WA} = \pi + \theta_{PA}, \quad r_{WA} = r_{PA}$$

- Beams are not round at most locations.
- Wires cannot be placed at every location.





Sextupole Doublet

Kick from a sextupole

$$\Delta x' = \frac{1}{2}S(x^2 - y^2), \quad \Delta y' = Sxy \qquad (3)$$

The kicks are even under a reflection

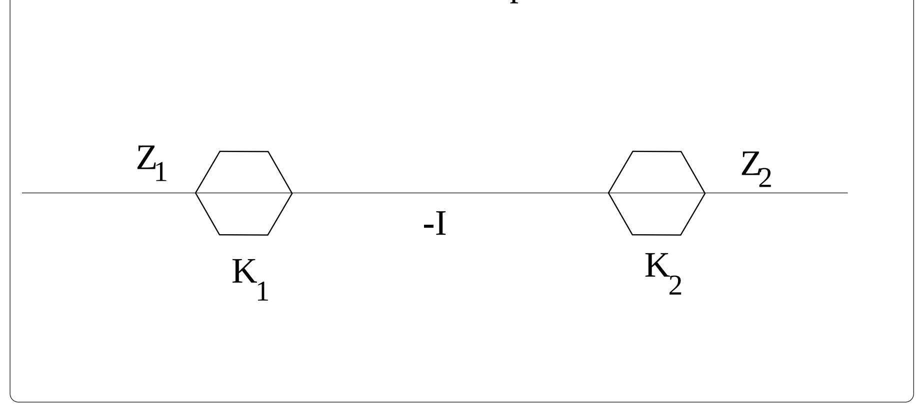
$$(x, y) \rightarrow (-x, -y)$$

Phase space vector $\vec{Z}$ is transformed as

$$\vec{Z}_2 = K_2 \odot (-I) \odot K_1 \odot \vec{Z}_1$$

while without the sextupoles, $\vec{Z}_2 = -\vec{Z}_1$.

Due to the symmetry property, the kicks cancel and we have no change in the phase space vector after the second sextupole,



Achromatic Sextupole Doublet

$$\vec{Z}_2 = -\vec{Z}_1$$

Caveats

- The sextupoles do influence the particle orbits in the region between them. Outside the doublet, the orbits do not change.
- The cancellation is not exact for off-momentum particles. The sextupoles act as momentum dependent quadrupoles.

Wire Compensation of 1 Kick

Principle of Compensation: The wire should restore the phase space trajectory to the point reached in the absence of the beam-beam interaction and the wire. The phase point after the wire is the same as though the motion were completely linear.

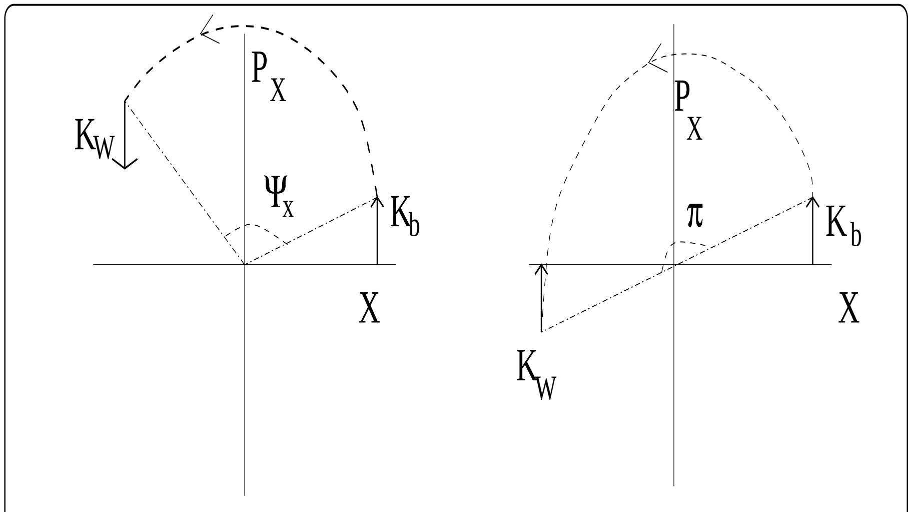
This requires

$$K_W \odot R(\psi_x, \psi_y) \odot K_b \odot \vec{Z}_i = R(\psi_x, \psi_y) \odot \vec{Z}_i \quad (4)$$

Assuming round beams and dropping the exponential part of the beam-beam kick, this condition can be met for *any particle* provided

$$\begin{aligned} \psi_x &= m_x \pi, \qquad \psi_y = m_y \pi \\ \frac{\beta_{y,W}}{\beta_{x,W}} &= \frac{\beta_{y,b}}{\beta_{x,b}} \\ I_W L &= ecN_p \end{aligned} \quad (5)$$

The compensation conditions also determine the transverse position of the wire.



Beam-beam kick, rotation, wire kick

Wire restores the phase pt. to original position

Wire Compensation of N Kicks

Compensation

$$K_W \odot R(\psi_{xn}, \psi_{yn}) \odot K_{b,n-1} \ldots R(\psi_{x1}, \psi_{y1}) \odot K_{b,1} \odot \vec{Z}_i = R(\sum \psi_x, \sum \psi_y) \odot \vec{Z}_i \qquad (6)$$

$\Rightarrow$ Phase advance between last beam-beam kick and the wire

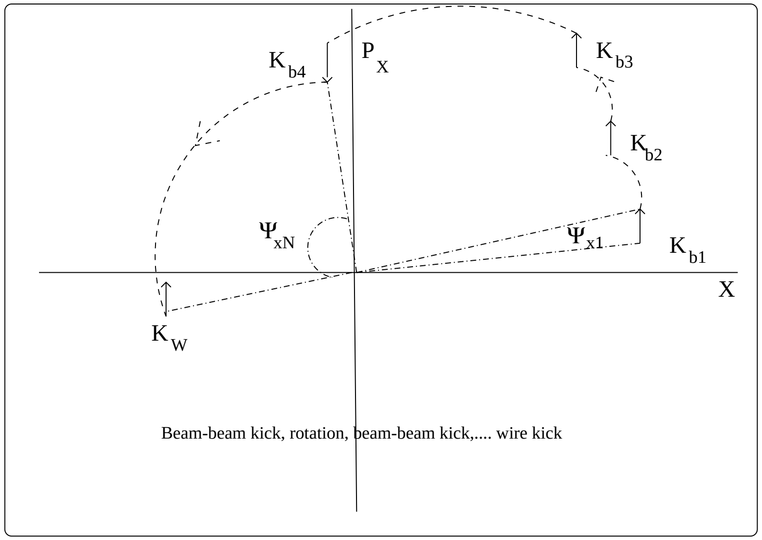
$$\tan \psi_{x,N} = -\frac{\sin[\Sigma_{i=1}^{N-1} \psi_{x,i}]\Delta P_{X,1} + \sin[\Sigma_{i=2}^{N-1} \psi_{x,i}]\Delta P_{X,2} + \ldots + \sin[\psi_{X,N-1}]\Delta P_{X,N-1}}{\left[\cos[\Sigma_{i=1}^{N-1} \psi_{x,i}]\Delta P_{X,1} + \cos[\Sigma_{i=2}^{N-1} \psi_{x,i}]\Delta P_{X,2} + \ldots + \cos[\psi_{x,N-1}]\Delta P_{X,N-1} + \Delta P_{X,N}\right]}$$

$$\tan \psi_{y,N} = -\frac{\sin[\Sigma_{i=1}^{N-1} \psi_{y,i}]\Delta P_{Y,1} + \sin[\Sigma_{i=2}^{N-1} \psi_{y,i}]\Delta P_{Y,2} + \ldots + \sin[\psi_{Y,N-1}]\Delta P_{Y,N-1}}{\left[\cos[\Sigma_{i=1}^{N-1} \psi_{y,i}]\Delta P_{Y,1} + \cos[\Sigma_{i=2}^{N-1} \psi_{y,i}]\Delta P_{Y,2} + \ldots + \cos[\psi_{y,N-1}]\Delta P_{Y,N-1} + \Delta P_{Y,N}\right]}$$

The total phase advance does not have to be π because the position also changes due to the successive beam-beam kicks.

Comments

- It is likely we will need to lump beam-beam kicks in blocks of $\pi, 2\pi, 3\pi, \ldots$ before compensation by a single wire.
- The cancellation can only work in an average sense.
- The transverse position of the wire is determined.



Tune shift compensation

Tune shifts at zero amplitude due to a wire

$$\Delta\nu_x = -\frac{\mu_0}{8\pi^2(B\rho)}\beta_x\left[\frac{I_W L\ \cos 2\theta_W}{r_W^2}\right], \quad \Delta\nu_y = +\frac{\mu_0}{8\pi^2(B\rho)}\beta_y\left[\frac{I_W L\ \cos 2\theta_W}{r_W^2}\right]$$

Both tune shifts have the same dependence on the wire parameters in square brackets $[\quad]$.

The wire also introduces coupling

$$\Delta\nu_{min} = \frac{\mu_0}{4\pi^2(B\rho)}\sqrt{\beta_x\beta_y}\left[\frac{I_W L\ \sin 2\theta_W}{r_W^2}\right]$$

The minimum tune split is of the same order as the tune shift.

The eigen tunes are

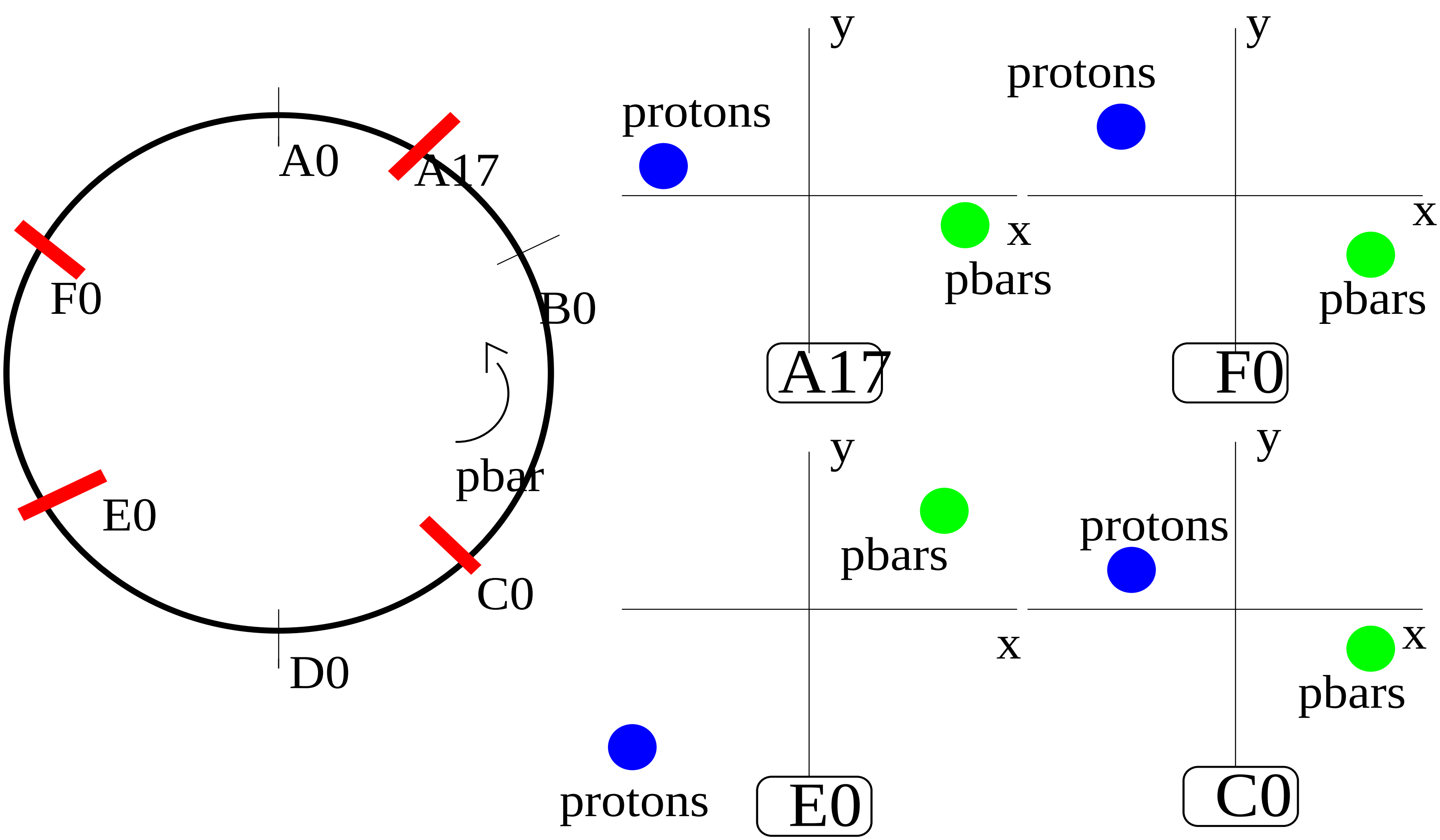
$$\nu_\pm = \frac{1}{2}(\nu_x+\nu_y) \pm \frac{1}{2}\Delta, \quad \Delta = [(\nu_x-\nu_y)^2 + \Delta\nu_{min}^2]^{1/2}$$

Use the wire to cancel the change in the eigen tunes

$$\Delta\nu_+ = \Delta\nu_{x,bb} + \Delta\nu_{x,W} + \frac{1}{2}\Delta(bb+wire) = 0, \quad \Delta\nu_- = \Delta\nu_{y,bb} + \Delta\nu_{y,W} - \frac{1}{2}\Delta(bb+wire) = 0$$

Comments:

- These can be used to find $I_W L/r_W^2$ and the angle θ_W
- For fixed beam-beam tune shifts, $I_W \propto r_W^2$. Grows faster with distance than the current required to cancel the kick itself where $I_W L \propto r_W$.
- The cancellation is effective exactly only at a single amplitude



Possible locations of 1m wire

Helix at wire locations (150 GeV)

Minimizing the norm of a map

Simplified version: Minimize the maximum phase space distortions by finding the appropriate corrector strengths.

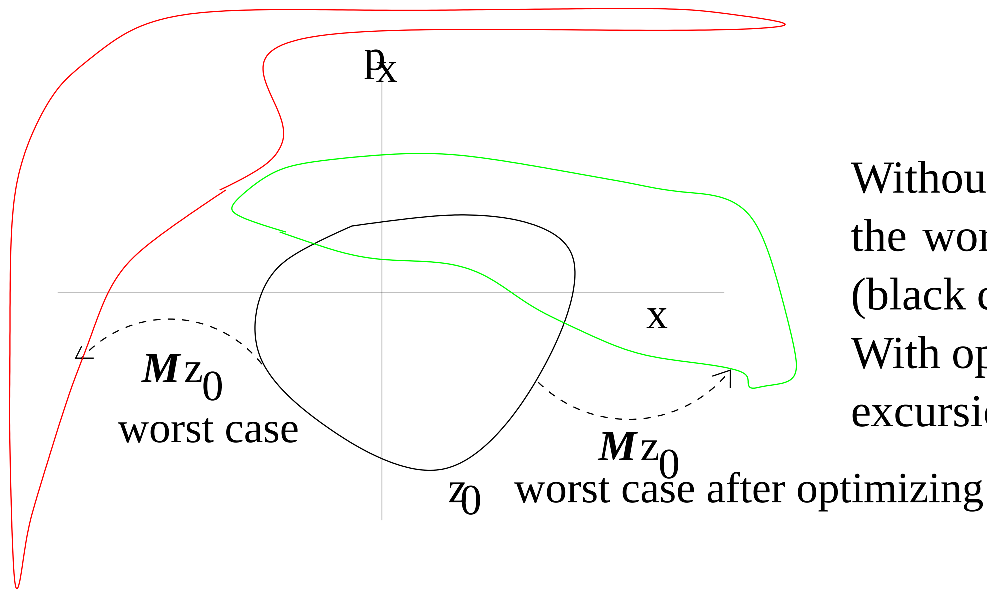
Consider 2D phase space and a map which transports particles

$$\vec{z}_f \equiv \begin{pmatrix} x_f \\ px_f \end{pmatrix} = \mathcal{M}\vec{z}_0 = \begin{pmatrix} \sum A_{ij} x_0^i px_0^j \\ \sum B_{ij} x_0^i px_0^j \end{pmatrix} \quad (7)$$

A_{ij}, B_{ij} are functions of the magnet and corrector strengths. The map is area-preserving which constrains the values of A, B.

Minimizing the norm of the map minimizes

$$|x_f| + |px_f| = \sum[|A_{ij}|x_0^i px_0^j + |B_{ij}|x_0^i px_0^j] \quad (8)$$



Without correction, nonlinearities could in the worst case transport the original domain (black curve) to the red curve.

With optimum corrector strengths, the largest excursions are limited to the green curve.

Results with 4 wires at Injection

Wire Parameters

Wire	$I\ [A]$	$x\ [mm]$	$y\ [mm]$
WA17	50	16.823	-1.033
WC0	75	12.993	-11.028
WE0	-25	21.232	14.282
WF0	232	14.004	-9.134

DA $[\sigma]$	**Number of turns**		
	10^4	10^5	10^6
Beam-beam on, no wires	**6.0**	4.0	**3.7**
Beam-beam on, best case wires	**7.0**	7.0	**5.5**

- DA not sensitive to ± 0.5 mm placement tolerance
- It is easy to make things worse, especially if the wire is too close to the beam
- **Next steps**
  - Optimize at collision, compensate the most damaging parasitics
  - Evaluate at injection with higher proton intensities
  - Evaluate possibility of wires outside the beam-pipe.